Chapter 1: Algebra Reference

1.1 Properties of Real Numbers and Polynomials

- Properties of Real Numbers
  - Use the following properties of real numbers: inverse and identity
  - Use the following properties of real numbers: commutative, associative, and distributive

- Polynomials
  - Add and subtract polynomials
  - Multiply binomials together
  - Multiply polynomials together
  - Perform operations with polynomials of several variables

1.2 Factoring

- Factoring Quadratics
  - Factor the greatest common factor of a polynomial
  - Factor a trinomial
  - Factor a trinomial by grouping
  - Factor a perfect square trinomial
  - Factor a difference of squares

- Other Factoring Techniques
  - Factor a cubic by grouping
  - Factor the sum and difference of cubes
  - Factor expressions using fractional or negative exponents
  - Factor expressions using greatest common factor and other technique

1.3 Rational Expressions

- Operations on Rational Expressions
  - Simplify rational expressions
  - Multiply rational expressions
  - Divide rational expressions
  - Add and subtract rational expressions

1.4 Equations

- Linear Equations
  - Identify identity, conditional, and inconsistent equations
  - Solve equations in one variable algebraically, variable just on one side
  - Solve equations in one variable algebraically, variable on both sides

- Quadratic Equations
  - Solve quadratic equations by factoring, leading coefficient 1
  - Solve quadratic equations by factoring, leading coefficient > 1
  - Solve quadratic equations by using the quadratic formula
Rational Equations
- Solve a rational equation, monomials in denominator
- Solve a rational equation, binomials in denominator
- Solve a rational equation, requires factoring to find least common denominator

1.5 Inequalities
- Linear Inequalities
  - Use interval notation
  - Use properties of inequalities
  - Solve simple inequalities in one variable algebraically
- Quadratic and Rational Inequalities
  - Solve quadratic inequalities in one variable, graph the solution set, and express the solution set using interval notation
  - Solve inequalities that involve rational expressions, graph the solution sets, and express the solution set using interval notation

1.6 Exponents
- Properties of Exponents
  - Understand exponent notation
  - Use the product rule of exponents
  - Use the quotient rule of exponents
  - Use the power rule of exponents
- Advanced Properties of Exponents
  - Use the negative and zero exponent rule
  - Find the power of a product
  - Find the power of a quotient
  - Simplify exponential expressions

1.7 Radicals
- Simplify Radicals
  - Evaluate square roots
  - Use the product rule to simplify square roots
  - Use the quotient rule to simplify square roots
- Operations with Radicals
  - Add and subtract square roots
  - Rationalize denominators with a monomial denominator
  - Rationalize denominators using the conjugate

Chapter 2: Linear Functions
2.1 Slopes and Equations of Lines
- Cartesian Coordinate System
  - Plot ordered pairs in a Cartesian coordinate system
  - Graph equations by plotting points
Identify Slopes and Intercepts
- Find the slope of a line given two points
- Understand the relationship between the slope and y-intercept of a line and its equation
- Find x-intercepts and y-intercepts

Finding Linear Equations
- Find equation of a line, in slope-intercept form, given slope and one point (point-slope formula)
- Find equation, in slope-intercept form, of a line passing through two given points
- Given slope and intercept, find the equation of a line and write it in standard form
- Find the equation of vertical and horizontal lines

Graphing Linear Equations
- Graph a linear equation using the slope and the origin

Parallel and Perpendicular Lines
- Given the equations of two lines, determine whether their graphs are parallel or perpendicular
- Write the equation of a line parallel to a given line
- Write the equation of a line perpendicular to a given line

2.2 Linear Functions and Applications
- Linear Functions
  - Understand function notation
  - Evaluate a linear function at a value
- Applications of Linear Functions
  - Solve supply and demand problems using linear functions
  - Solve cost analysis problems using linear functions
  - Solve break even analysis problems using linear functions

2.3 The Least Squares Line
- The Least Squares Line
  - Find the linear regression equation given a list of data points
  - Make predictions using a line of best fit
  - Find and interpret the correlation coefficient

Chapter 3: Systems of Linear Equations and Matrices
3.1 Solving Linear Systems
- Linear Systems in Two Variables
  - Solve systems of equations in two variables by graphing
  - Solve systems of equations in two variables by substitution
  - Solve systems of equations in two variables by addition
  - Identify inconsistent and dependent systems of equations containing two variables, and express the solution of dependent equations
• Linear System in Three Variables
  - Determine whether an ordered triple is a solution to a system
  - Solve systems of three equations in three variables
  - Identify inconsistent and dependent systems of equations containing three variables, and express the solution of a system of dependent equations

• Applications of Linear Systems
  - Use systems of equations to investigate profits
  - Write and solve a system of equations in two variables from a word problem

3.2 Solving Linear Systems by the Gauss-Jordan Method
• Solving Systems with Gaussian Elimination
  - Convert between a system of equations and its corresponding augmented matrix
  - Use row operations to solve a system of linear equations in two variables
  - Use row operations to solve a system of linear equations in three variables
  - Use matrices to solve applications of systems of linear equations

3.3 Operations with Matrices
• Addition and Subtraction of Matrices
  - Determine the order of a matrix and describe elements within a matrix
  - Add or subtract matrices

• Multiplication of Matrices
  - Multiply a matrix by a scalar
  - Find the sum or difference of scalar multiples
  - Multiply two matrices

3.4 Matrix Inverses and Determinants
• Determinants of Matrices
  - Find the determinant of a 2x2 matrix
  - Find the determinant of a 3x3 matrix

• Inverse and Identity Matrices
  - Understand the identity matrix and how it relates to the inverse matrix
  - Determine if a matrix is invertible using the determinant
  - Find the inverse of a 2x2 matrix
  - Find the inverse of a 3x3 matrix

• Solving Systems with Inverses
  - Solve a system of linear equations using the inverse of a 2x2 matrix
  - Solve a system of linear equations using the inverse of a 3x3 matrix

3.5 Input-Output Models
• Input-Output Matrices
  - Create an input output matrix for a given application
  - Calculate the amount of commodities produced given an input output matrix and a production matrix
• Determine the production matrix that will satisfy a given demand matrix
• Find the production of a commodity in a closed input output model

Chapter 4: Linear Programming - The Graphical Method
4.1 Graphing Linear Inequalities
  • Graphs of Linear Inequalities
    • Solve a linear inequality in two variables by graphing
    • Solve a linear system of inequalities by graphing
4.2 Solving Linear Programming Problems Graphically
  • Solving Linear Programming Problems Graphically
    • Graph a feasible region given a set of constraints
    • Find the maximum value of an objective function given constraints by graphing
4.3 Applications of Linear Programming
  • Applications of Linear Programming
  • Solve application problems using linear programming

Chapter 5: Linear Programming - The Simplex Method
5.1 Slack Variables and the Pivot
  • Finding Solutions using Initial Simplex Tableaus
    • Rewrite a linear programming problem using slack variables and create an initial simplex tableau
    • Read a solution from an initial simplex tableau
    • Find a new solution by pivoting an initial simplex tableau
5.2 Maximization and Minimization Problems
  • Solving Maximization Problems with the Simplex Method
    • Solve maximization problems using the simplex method
  • Transposing a Matrix and Finding the Dual of a Linear Programming Problem
    • Determine the transpose of a matrix
    • Determine the dual of a linear programming problem
  • Solving Minimization Problems with Duality
    • Solve minimization problems using the theorem of duality
5.3 Nonstandard Problems
  • Solving Nonstandard Problems
    • Solve a nonstandard linear programming problem
    • Solve a nonstandard linear programming application problem
Chapter 6: Mathematics of Finance

6.1 Simple Interest
- Simple Interest
  - Calculate simple interest
  - Calculate interest discounts on a discounted loan

6.2 Compound Interest
- Compound Interest
  - Calculate periodically compounded interest
  - Calculate compound interest
  - Calculate continuously compounded interest
  - Calculate effective annual yield

6.3 Annuities, Stocks, and Bonds
- Annuities
  - Calculate the value of an annuity
  - Calculate the payment needed to achieve a determined future value
- Stocks
  - Define stock terminology
  - Read a stock table

6.4 Installment Loans, Amortization, and Credit Cards
- Mortgages and Loans
  - Calculate the monthly payment and interest cost for a mortgage
  - Construct a loan amortization schedule
  - Choose the best installment loan plan
- Credit Cards
  - Recognize key features of credit cards
  - Calculate the average daily balance of a credit card
  - Determine interest to be paid on a card's next billing date

Chapter 7: Logic

7.1 Statements and Logical Connectives
- The Building Blocks of Logic
  - Identify and negate simple statements
  - Identify and negate quantified statements
- Symbolic Representation of Statements
  - Identify logical connectives and compound statements
  - Represent and/or/not statements in symbolic form and English
- Conditional Statements
  - Represent conditional statements in symbolic form and English
  - Write biconditional statements in symbolic form and English
  - Represent symbolic statements with parentheses using dominance of connectives
7.2 Truth Tables for Negation, Conjunction, and Disjunction

- Introduction to Truth Tables
  - Construct a truth table for a statement with a conjunction and/or a negation and determine its truth value
  - Construct a truth table for a statement with a disjunction and/or a negation and determine its truth value
  - Construct a truth table for a compound statement with a conjunction and disjunction and determine its truth value

7.3 Truth Tables for the Conditional and Biconditional

- Truth Tables for Conditional and Biconditional Statements
  - Construct a truth table for a conditional statement and determine its truth value
  - Construct a truth table for a biconditional statement and determine its truth value

- Self-Contradictions, Tautologies, and Implications
  - Identify self-contradictions, tautologies, and implications

7.4 Equivalent Statements

- Equivalent Statements and De Morgan's Equivalence Laws
  - Determine if two symbolic statements are equivalent using a truth table
  - Determine if two statements given in English are equivalent using a truth table
  - Determine if two statements are equivalent using De Morgan's laws

- Conditional States and Equivalence
  - Convert a disjunction into an equivalent conditional statement
  - Determine if two conditional statements are equivalent

7.5 Symbolic Arguments

- Drawing and Verifying Conclusions
  - Draw a conclusion from a conditional statement
  - Determine if an argument is valid using a truth table
  - Identify and validate the standard forms of arguments

7.6 Euler Diagrams and Syllogistic Arguments

- Euler Diagrams and Syllogistic Arguments
  - Identify syllogistic arguments
  - Represent a syllogistic argument with a Euler diagram
  - Determine if a syllogistic argument is valid with a Euler diagram

7.7 Switching Circuits

- Switching Circuits and Symbolic Logic
  - Convert between symbolic statements and switching circuits
  - Determine conditions for when a lightbulb will be turned on in a switching circuit
  - Determine if two switching circuits are equivalent
Chapter 8: Sets and Counting Principles

8.1 Set Concepts

- Introduction to Sets and Set Builder Notation
  - Represent a set using a written description and the roster method
  - Represent a set using set builder notation
- Set Equivalence
  - Identify the cardinal number for a set
  - Determine if two sets are equivalent
  - Determine if two sets are equal
- Types of Sets
  - Identify subsets, universal sets, and empty sets
  - Distinguish between finite and infinite sets
- Subsets and Proper Subsets
  - Identify subsets and proper subsets using set notation
  - Determine the number of subsets and proper subsets in a given set

8.2 Venn Diagrams and Set Operations

- Representing Sets with Venn Diagrams
  - Illustrate the universal set, a set, and complement of a set using a Venn diagram
  - Illustrate two sets using Venn diagram and set notation
- Set Relationships
  - Determine the complement of a set using Venn diagrams and proper set notation
  - Determine the intersection of two sets using Venn diagrams and set notation
  - Determine the union of two sets using Venn diagrams and set notation
- Set Operations
  - Perform operations on sets
  - Find the difference and cartesian product of two sets
  - Use Venn diagrams to find the result of set operations on two sets
  - Determine the cardinal number of a union of two finite sets

8.3 Venn Diagrams with Three Sets and Verification of Equality of Sets

- Construct a Venn Diagram of Three Sets
  - Perform set operations on three sets
  - Represent three sets using Venn diagrams

The Fundamental Counting Principle

- The Fundamental Counting Principle
  - Solve counting problems using the addition principle
  - Solve counting problems using the multiplication principle

8.4 Permutations and Combinations

- Permutations
  - Evaluate an expression with factorials
  - Find the number of permutations of n distinct objects using the multiplication principle
Chapter 9: Probability

9.1 Introduction to Probability

- Sample Spaces and Events
  - Determine the sample space of an experiment
  - Determine an event of an experiment

- Fundamentals of Probability
  - Compute the probability of equally likely outcomes
  - Compute the probability of equally likely outcomes in application

- Probability with Permutations and Combinations
  - Compute probability involving permutations
  - Compute probability involving combinations

- The Complement Rule and Probability
  - Use the complement rule to compute probabilities
  - Compute the probability of an event happening at least once

- Odds and Expected Value
  - Compute the expected value of an event
  - Compute odds using probability

9.2 Conditional Probability and Independent Events

- Independent Events
  - Compute the probability of the union of two events
  - Compute the probability of two independent events occurring

- Dependent Events and Conditional Probability
  - Compute the conditional probability of a dependent event occurring
  - Compute the probability of two or more dependent events occurring

9.3 Binomial Probability

- Binomial Experiments
  - Identify a binomial experiment
  - Determine the binomial probability of success in an experiment performed multiple times
  - Calculate expected value for binomial probability

9.4 Bayes’ Theorem

- Bayes’ Theorem
  - Apply Bayes’ theorem to solve an application problem
9.5 Random Variables, Probability Distributions and Expected Value

- Random Variables, Probability Distributions, and Expected Value
  - Calculate probability distribution
  - Calculate the expected value of a random variable

Chapter 10: Statistics

10.1 Sampling, Frequency Distributions, and Graphs

- Sampling and Parameters
  - Understand the definitions of population, sampling, statistic, parameter, and data
  - Identify stratified, cluster, systematic, and convenience sampling
  - Identify sampling errors and bias
  - Identify situations in which statistics can be misleading

- Frequency Distributions and Histograms
  - Construct and understand frequency tables for a set of data
  - Create and interpret histograms
  - Create and interpret stem-and-leaf plots

10.2 Measures of Central Tendency and Measures of Dispersion

- Means and Medians
  - Find the mean of a set of data
  - Find the mean from a frequency table
  - Find the median of a set of data

- Modes, Midranges, and Choosing a Measurement
  - Find the mode of a set of data
  - Find the midrange of a set of data
  - Determine whether the mean, median, or mode is the best measure of center for a data set

- Standard Deviation
  - Compute the sample variance and sample standard deviation
  - Interpret the standard deviation of a set of data

10.3 The Normal Distribution, Margins of Error, and Skewness

- The Normal Distribution
  - Understand the notation and interpret the parameters of a normal distribution
  - Compute z-scores and use them to compare values from different data sets
  - Determine if a data set is skewed

- Percentiles, Quartiles, and Margins of Error
  - Find and interpret percentiles and quartiles of a data set
  - Calculate and interpret margin of error

- Problem Solving with the Normal Distribution
  - Standardize a normally distributed random variable
  - Calculate the mean and standard deviation of a standard normal distribution
Chapter 11: Nonlinear Functions

11.1 Properties of Functions

- Functions and Function Notation
  - Identify domain and range from a set of ordered pairs
  - Determine whether a relation represents a function
- Relations and Functions
  - Determine whether a function is one-to-one
  - Use the vertical line test to identify functions
  - Use the horizontal line test to identify one-to-one functions

11.2 Quadratic Functions

- Characteristics of Parabolas
  - Determine axis of symmetry and vertex of parabolas from a graph
  - Determine x- and y-intercepts of parabolas from a graph
- Graphs of Quadratic Functions
  - Find the direction a parabola opens and its axis of symmetry and vertex from the general form of its equation
  - Identify the axis of symmetry and vertex of a parabola from its equation in standard form
  - Write the equation of a quadratic function given vertex and a point on a graph
  - Write the equation of a quadratic function given intercepts on a graph
  - Write the equation of a quadratic function in standard form given the equation in general form
- Applications of Quadratic Functions
  - Find the domain and range of a quadratic function
  - Determine the maximum and minimum values of quadratic functions
  - Find the x- and y-intercepts of a quadratic function

11.3 Transformations of Functions

- Transformations of Functions
  - Graph functions using vertical and horizontal shifts
  - Graph functions using reflections about the x-axis and the y-axis
  - Graph functions using compressions and stretches
  - Combine transformations

11.4 Polynomial and Rational Functions

- Polynomial Functions
  - Identify power functions and polynomial functions
  - Graph polynomial functions
  - Write a formula for a polynomial function from a graph
  - Determine equation of a polynomial given key information
Asymptotic Behavior of Rational Functions
- Use arrow notation to describe local behavior and end behavior of rational functions
- Identify vertical asymptotes and removable discontinuities of rational functions
- Identify horizontal and slant asymptotes of rational functions

Graphs and Applications of Rational Functions
- Find the intercepts of a rational function
- Graph rational functions
- Find the equation of a rational function from a graph

11.5 Exponential and Logarithmic Functions
- Identify and Evaluate Exponential Functions
  - Identify exponential functions
  - Evaluate exponential functions
  - Calculate continuous growth and decay
- Graphing Exponential Functions
  - Graph exponential functions
  - Graph exponential functions using transformations
- Relate Logarithms and Exponents
  - Convert from logarithmic to exponential form
  - Convert from exponential to logarithmic form
- Evaluate Logarithmic Expressions
  - Evaluate logarithms with positive integer solutions
  - Evaluate logarithms with negative integer solutions
  - Use common logarithms
  - Use natural logarithms
- Basic Properties of Logarithms
  - Understand the basic properties of logarithms
  - Use the product rule for logarithms
  - Use the quotient rule for logarithms
  - Use the power rule for logarithms
- Solve Logarithmic Equations
  - Use the definition of a logarithm to solve logarithmic equations
  - Use logarithm properties and the definition of the logarithm to solve logarithmic equations
  - Use the one-to-one property of logarithms to solve logarithmic equations
- Applications of Exponential and Logarithmic Functions
  - Model exponential growth
  - Model exponential decay
  - Applied logarithmic models
  - Choose an appropriate model for data
  - Express an exponential model in base e
Chapter 12: Markov Chains

12.1 Properties of Markov Chains

- Transitions
  - Identify transition diagrams and transition matrices
  - Create a transition diagram and matrix for a given word problem
- States
  - Find the second state of a system given a transition matrix and initial state
  - Find powers of a transition matrix
  - Solve application problems using powers of transition matrices

12.2 Regular Markov Chains

- Regular Transition Matrices and Stationary Matrices
  - Determine if a transition matrix is regular
  - Find a stationary matrix for a given transition matrix
  - Solve application problems using stationary matrices

12.3 Absorbing Markov Chains

- Absorbing States
  - Find absorbing states given a transition matrix
  - Determine if a transition matrix is for an absorbing Markov chain
  - Write a transition matrix in standard form
- Limiting Matrix
  - Find the limiting matrix for an absorbing Markov chain

Chapter 13: Limits and the Derivative

13.1 Introduction to Limits

- Limits From a Graph or Table
  - Understand the limit of a function and evaluate a limit from a table
  - Evaluate limits graphically
  - Understand the properties of limits
- Limits Analytically for Continuous and Piecewise Functions
  - Evaluate two-sided limits analytically for continuous functions
  - Evaluate limits analytically for piecewise functions
  - Evaluate limits analytically for absolute value functions
- Limits Analytically for Functions with Removable Discontinuities
  - Evaluate two-sided limits analytically for rational functions with removable discontinuities by factoring
  - Evaluate two-sided limits analytically for rational functions with removable discontinuities through expansion
  - Evaluate two-sided limits analytically for complex fractions with removable discontinuities
  - Evaluate two-sided limits analytically for rational functions that contain radicals with removable discontinuities
13.2 Infinite Limits and Limits at Infinity

- Infinite Limits
  - Evaluate limits analytically for functions with essential discontinuities
- Limits at Infinity
  - Evaluate limits of polynomial functions at infinity
  - Evaluate limits of rational functions at infinity
  - Evaluate limits of radical and exponential functions at infinity

13.3 Continuity

- Continuity and the Intermediate Value Theorem
  - Understand the definition of continuity
  - Understand types of discontinuity in rational functions
  - Understand and apply the intermediate value theorem
- Continuity of Piecewise Functions
  - Determine whether a piecewise function is continuous
  - Determine the value that makes a piecewise function continuous
- Rational and Quadratic Inequalities
  - Solve quadratic inequalities in one variable, graph the solution set, and express the solution set using interval notation
  - Solve inequalities that involve rational expressions, graph the solution sets, and express the solution set using interval notation

13.4 The Derivative

- Secant Lines and Average Rates of Change
  - Find the average rate of change given a function
  - Find the average rate of change given a table or graph
  - Find the average rate of change given a function and variable intervals
- Tangent Lines and Instantaneous Velocities
  - Determine the sign of the slope of a line tangent to a function at a given point
  - Estimate the slope of the line tangent to a point on a curve
  - Estimate the instantaneous rate of change of a function from successively closer approximations
- The Definition of the Derivative
  - Understand the limit definition of the derivative
  - Use the limit definition to find the derivative of a polynomial function
  - Use the limit definition to find the derivative of a rational function
  - Use the limit definition to find the derivative of a function with a radical

13.5 Basic Differentiation Properties

- The Power Rule and the Sum and Difference Rules
  - Use the constant, constant multiple, and power rule for monomials
  - Apply the sum and difference rules to combine derivatives
● Use the Power Rule to Explore Tangent Lines
  ● Find the equation of the line tangent to a polynomial at a point
  ● Determine where a function has a horizontal tangent
  ● Determine the points on a function when tangent lines have a given slope

13.6 Differentials
● Differentials and Finding Error
  ● Compute a differential
  ● Estimate the amount of propagated and relative error using differentials

13.7 Marginal Analysis in Business and Economics
● Marginal Cost and Revenue
  ● Compute and interpret marginal cost
  ● Compute and interpret revenue and marginal revenue
● Marginal Average Cost and Revenue
  ● Compute average cost and revenue
  ● Compute and interpret marginal average cost

Chapter 14: Additional Derivative Topics
14.1 Derivatives of Exponential and Logarithmic Functions
● Derivatives of Exponential Functions with Base e
  ● Find the derivative of an exponential function with base e
  ● Find the derivative of an exponential function with any base
● Derivatives of Natural Log Functions
  ● Find the derivative of a natural logarithmic function
  ● Use properties of logarithms to find the derivative of a natural logarithmic function
● Derivatives of Logarithmic Functions of Any Base
  ● Find the derivative of a logarithmic function that is not base e
  ● Use properties of logarithms to find the derivative of a logarithmic function that is not base e
● Logarithmic Differentiation
  ● Use logarithmic differentiation
  ● Use logarithmic differentiation with all properties of logarithms
14.2 Derivatives of Products and Quotients
● The Product and Quotient Rules
  ● Use the product rule to find the derivative of a function in the form f(x)g(x)
  ● Use the quotient rule to find the derivative of a function in the form f(x)/g(x)
  ● Use the product rule to find the derivative of a function in the form f(x)g(x)h(x)
  ● Combine the product and quotient rules with polynomials
• The Product and Quotient Rules with Exponential Functions
  • Use product and quotient rules to find the derivative of exponential functions with base e
  • Use product and quotient rules to find the derivative of exponential functions with any base

14.3 The Chain Rule
• Using the Chain Rule
  • Use the chain rule with the power rule
  • Use the chain rule with the product and quotient rules

14.4 Implicit Differentiation
• Use Implicit Differentiation
  • Use implicit differentiation
  • Use implicit differentiation and the product rule
  • Use implicit differentiation to find the equation of a tangent line

14.5 Related Rates
• Related Rates for Volume or Area Problems
  • Use related rates to solve problems involving volume
  • Use related rates to solve problems involving area

• Related Rates in Other Applications
  • Use related rates to solve problems involving distance
  • Use related rates to solve problems involving angles or shadows

14.6 Elasticity of Demand
• Relative Rate of Change and Elasticity of Demand
  • Find the relative and percentage rate of change of a function
  • Compute elasticity of demand
  • Interpret elasticity of demand

Chapter 15: Graphing and Optimization
15.1 First Derivative and Graphs
• First Derivative Test
  • Understand the relationship between the graph of a function and the sign of its derivative
  • Use the first derivative test to find local extrema from a graph
  • Use the first derivative test to find local extrema given a function

• The Graph of the Derivative Function
  • Estimate the value of a derivative at a point on a graph using a tangent line
  • Determine the open intervals where the first derivative is positive or negative from a graph
  • Determine the graph of the derivative function given the graph of a polynomial function
15.2 Second Derivative and Graphs
- Concavity and the Second Derivative Test
  - Determine concavity and find the inflection points from a graph of f(x)
  - Determine concavity and find the inflection points given a function
  - Use the second derivative test to find local extrema given a function

15.3 L'Hospital's Rule
- Using L'Hospital's Rule
  - Apply L'Hospital's Rule in the 0/0 case
  - Apply L'Hospital's Rule in the (infinity/infinity) case
  - Determine when to apply L'Hospital's Rule

15.4 Curve Sketching Technique
- Sketch the Curve of a Function
  - Sketch the graph of a polynomial
  - Sketch the graph of a rational function
  - Sketch the graph of a function with a cusp

15.5 Absolute Maxima and Minima
- Extreme Value Theorem and Absolute Extrema
  - Understand the extreme value theorem
  - Locate local and absolute extrema from a graph
  - Locate critical points using derivatives
  - Locate absolute extrema

15.6 Optimization
- Applied Optimization Problems
  - Maximize or minimize area or volume
  - Minimize travel time
  - Maximize revenue
  - Minimize surface area
- Optimization Problems in the Abstract
  - Maximize the area of an inscribed rectangle
  - Maximize and minimize quantities given an expression with two variables
  - Minimize distance of a function to a point
- Optimization of Lot Size and Quantity
  - Find the economic lot or economic order quantity size which minimizes total cost

Chapter 16: Integration
16.1 Antiderivatives and Indefinite Integrals
- Antiderivatives and the Integral
  - Find the antiderivative of a function
  - Understand integral notation and verify an indefinite integral
- Understand the properties of indefinite integrals
- Evaluate indefinite integrals involving constants or powers

16.2 Integration by Substitution
- Substitution and the Power Rule
  - Use substitution to find an indefinite integral with the power rule
  - Use substitution to evaluate a definite integral with the power rule

16.3 Differential Equations; Growth and Decay
- Basics of Differential Equations
  - Verify a solution of a differential equation
  - Identify the order of a differential equation
- Linear Differential Equations
  - Find a general solution to a linear differential equation
  - Find a particular solution to a linear differential equation
- Initial Value Problems
  - Verify a solution to a differential equation initial value problem
  - Solve a differential equation initial value problem
  - Solve applications of differential equation initial value problems
- Creating Direction Fields
  - Create a direction field for a first-order differential equation
  - Sketch a solution curve given a direction field
- Euler's Method
  - Use Euler's method to approximate the solution of a differential equation
- Differential Equations for Growth and Decay
  - Model and solve growth and decay applications with differential equations

16.4 The Definite Integral
- Left and Right Riemann Sums
  - Approximate the area under a curve using left-endpoint approximation
  - Approximate the area under a curve using right-endpoint approximation
- Midpoint and Trapezoid Rule
  - Approximate the area under a curve using midpoint approximation
  - Approximate the area under a curve using trapezoidal approximation
- Defining Definite Integrals
  - Explain the terms integrand, limits of integration, and variable of integration, and describe when a function is integrable
  - Evaluate an integral using the definition of the definite integral and left- or right-endpoint approximations
- Calculating Definite Integrals with a Geometric Approach
  - Use a geometric formula to calculate a definite integral
  - Calculate net signed areas under a line using formulas for area of a triangle
  - Calculate total area under a function using geometric formulas
• Properties of the Definite Integral
  ● Use the properties of the definite integral

16.5 The Fundamental Theorem of Calculus
• Integrals and Derivatives with the Fundamental Theorem of Calculus
  ● Use the Fundamental Theorem of Calculus to find the derivative of an integral function
  ● Use the Fundamental Theorem of Calculus and the chain rule to find a derivative
  ● Use the Fundamental Theorem of Calculus with two variable limits of integration
• Evaluating Definite Integrals with the Fundamental Theorem of Calculus
  ● Evaluate definite integrals with the Fundamental Theorem of Calculus for functions with positive integer exponents
  ● Evaluate definite integrals with the Fundamental Theorem of Calculus for functions with rational exponents
  ● Evaluate definite integrals with the Fundamental Theorem of Calculus and the power rule by simplifying
• Average Value of a Function
  ● Find the average value of a function over an interval

Chapter 17: Additional Integration Topics
17.1 Area Between Curves
• Finding the Area of a Region Bounded by Two Curves
  ● Find the area of a region between two linear functions
  ● Find the area of a region bounded between a linear function and another function
  ● Find the area of a region bounded between two curves
• Finding the Area of Compound Regions
  ● Find the area of a region bounded by two functions that cross
  ● Find the area of a region bounded above by two different functions
• Applications of Area Between Curves
  ● Find and interpret the Gini index

17.2 Volume of Revolution
• Volume Using the Disk Method
  ● Use the disk method to find the volume of a solid of revolution around the x-axis with polynomials or roots

17.3 Applications in Business and Economics
• Income Streams
  ● Find the total income for a continuous stream
  ● Find the present value of a continuous stream
  ● Find the future value of a continuous stream
17.4 Integration by Parts
- Basic Integration by Parts with Indefinite Integrals
  - Use integration by parts when u and v are given
  - Use integration by parts for indefinite integrals

17.5 Integration Using Tables
- Integration Tables
  - Use a formula from an integration table to evaluate an integral
- Integration with Computer Algebra Systems
  - Use a computer algebra system to evaluate an integral
- Reduction Formulas
  - Use a table with reduction formulas to evaluate an integral

Chapter 18: Multivariable Calculus
18.1 Functions of Several Variables
- Introduction to Multivariable Functions
  - Evaluate a multivariable function
  - Sketch a point in three-dimensional space
  - Graph cross sections of a multivariable function

18.2 Partial Derivatives
- Partial Derivatives of a Function of Two Variables
  - Find the partial derivative of a function of two variables
  - Estimate the partial derivative of a function at a point from a graph or contour map
- Total Differential
  - Use the differential to approximate the change in a function given the change in the inputs or to calculate maximum error
- Partial Derivatives of a Function of Three or More Variables
  - Find the partial derivative of a function of three variables
- Higher Order Partial Derivatives
  - Find the higher order partial derivatives of a function of two variables
- The Chain Rule for Functions of Several Variables
  - Use the chain rule for one independent variable
  - Use the chain rule for two independent variables
  - Use the generalized chain rule
18.3 Maxima and Minima
- Critical Points and the Second Derivative Test for Functions of Two Variables
  - Find critical points of a function of two variables
  - Use the second derivative test to classify critical points of a function of two variables
- Absolute Extrema and Applications for Functions of Two Variables
  - Find the absolute extrema of a function of two variables on a closed region
  - Solve maximization and minimization word problems with multiple variables

18.4 Maxima and Minima Using Lagrange Multipliers
- Lagrange Multipliers with One Constraint
  - Use Lagrange multipliers to find maximum and minimum values of a function of two variables with a single constraint
  - Use Lagrange multipliers to find maximum and minimum values of a function of three variables with a single constraint
- Lagrange Multipliers with Two Constraints
  - Use Lagrange multipliers to find maximum and minimum values of a function with two constraints

18.5 Double Integrals over Rectangular Regions
- Iterated Integrals and Properties of Double Integrals
  - Recognize and use some of the properties of double integrals
  - Evaluate a double integral over a rectangular region by writing it as an iterated integral
  - Evaluate a double integral over a rectangular region by reversing the order of integration
- Applications of Double Integrals Over Rectangular Regions
  - Find the volume under a surface
  - Find the average value of a function over a rectangular region

18.6 Double Integrals over More General Regions
- Double Integrals Over Nonrectangular Regions
  - Recognize when a function of two variables is integrable over a general region
  - Evaluate a double integral by computing an iterated integral over a region bounded by two lines and two functions
- Double Integrals by Decomposing Regions or Changing the Order of Integration
  - Evaluate a double integral over a more complex region by decomposing the region
  - Simplify the calculation of an iterated integral by changing the order of integration
- Applications of Double Integrals Over General Regions
  - Use double integrals to calculate the area of a general plane region
  - Use double integrals to calculate the volume of a region between two surfaces over a general plane region
  - Find the average value of a function over a general region
Chapter 19: Trigonometric Functions

19.1 Trigonometric Functions Review

- Angles as Rotations and Radian Measures
  - Identify the measure of positive and negative angles in standard position and the quadrant of the terminal side
  - Convert between degree and radian measure of an angle
  - Understand when two angles are coterminal

- The Six Trigonometric Ratios
  - Use right triangles to evaluate sin, cos, and tan functions
  - Evaluate reciprocal trig functions using right triangles or a sin, cos, or tan function

- Sine and Cosine Values in the First Quadrant
  - Understand sin and cos values on the unit circle
  - Find exact sin and cos values for angles in the first quadrant of the unit circle

- Sine and Cosine Values with Reference Angles and a Calculator
  - Find the reference angle for a given angle
  - Use reference angles to evaluate sin and cos functions
  - Use reference angles to find coordinates on the unit circle
  - Evaluate sin and cos functions with a calculator

- The Other Trigonometric Ratios on the Unit Circle
  - Find the sec, csc, tan, and cot values for angles in the first quadrant of the unit circle
  - Use reference angles to evaluate sec, csc, tan, and cot functions
  - Evaluate trigonometric functions with a calculator

- Use Given Trigonometric Ratios to Find Other Ratios
  - Understand the relationship between the quadrant in which an angle falls and the signs of the trig functions of that angle
  - Use the pythagorean identity
  - Find the values of all trigonometric functions given coordinates on a unit circle
  - Find the values of all trigonometric functions given the value of one trigonometric function

- Characteristics of Sin and Cos Graphs
  - Graph the sin function and understand its properties
  - Graph the cos function and understand its properties

- Applications of Trigonometric Functions
  - Use sinusoidal functions to solve real-world applications

19.2 Derivatives of Trigonometric Functions

- Derivatives with Trigonometric Functions
  - Find the derivative of a function with sine or cosine
  - Use the product or quotient rule to find a derivative with sine or cosine
  - Use the chain rule with trigonometric functions

- Applications of Trigonometric Derivatives
  - Compute derivatives of trigonometric functions in application problems
19.3 Integration of Trigonometric Functions

- Integration with Trigonometric Functions
  - Evaluate indefinite integrals involving trigonometric functions
  - Evaluate definite integrals involving trigonometric functions
  - Compute integrals of trigonometric functions in application problems

Chapter 20: Differential Equations

20.1 Separation of Variables

- Finding Differential Equation Solutions using Separation of Variables
  - Find a general solution to a differential equation composed of two polynomials using separation of variables
  - Find a specific solution to a differential equation composed of two polynomials using separation of variables
- Application Problems using Separation of Variables
  - Solve solution concentration problems using separation of variables
  - Solve Newton's law of cooling problems using separation of variables

20.2 First-Order Linear Differential Equations

- Recognizing and Solving First-order Linear Differential Equations
  - Identify first-order linear differential equations
  - Write first-order linear differential equations in standard form
  - Solve a first-order linear differential equation using an integrating factor
- Applications of First-order Linear Differential Equations
  - Solve first-order differential equation problems about compound interest
  - Solve first-order differential equation problems involving equilibrium price

Chapter 21: Taylor Polynomials and Sequences and Series

21.1 Taylor Polynomials

- Taylor and Maclaurin Polynomials
  - Recognize a Taylor series
  - Find the Taylor polynomials for a function at a value

21.2 Taylor Series

- Representing Functions with Taylor and Maclaurin Series
  - Represent a function at a value with a Taylor series and determine the interval of convergence
  - Find the Maclaurin series for a function and show that the series converges

21.3 Operations on Taylor Series

- Finding the Maclaurin Series that Represents a Function
  - Find the Maclaurin series for a trigonometric function
  - Find the Maclaurin series for a logarithmic or exponential function
  - Find a Maclaurin series by differentiating another series
21.4 Approximations Using Taylor Series

- Estimating Function Values with Taylor and Maclaurin Series
  - Find the Maclaurin polynomials for a function
  - Estimate a function value using Taylor polynomials
  - Determine the error of an estimated function value using Taylor’s theorem
  - Estimate a trigonometric function value using Maclaurin polynomials
- Alternating Series and the Alternating Series Test
  - Determine if an alternating series converges or diverges using the alternating series test
  - Estimate the remainder of an alternating series
  - Determine whether a series converges absolutely or conditionally

21.5 Sequences and Series

- Geometric Sequences
  - Find the common ratio of a geometric sequence
  - Write terms of a geometric sequence
  - Write a recursive formula for a geometric sequence
  - Write an explicit formula for a geometric sequence
- Applications of Series
  - Solve application problems with arithmetic series
  - Solve application problems with geometric series
  - Find the equivalent fraction for a repeating decimal
  - Solve an annuity problem
- Finite and Infinite Geometric Series
  - Find the sum of a finite geometric series
  - Determine if the sum of an infinite series is defined
  - Find the sum of an infinite geometric series
- Newton's Method
  - Use Newton's method to approximate the root of a polynomial
  - Use Newton's method to approximate a square root
  - Determine when Newton's method does not work

Chapter 22: Probability and Calculus

22.1 Improper Integrals

- Improper Integrals over Infinite Intervals
  - Evaluate an improper integral over an infinite interval
  - Evaluate an improper integral from negative infinity to positive infinity

22.2 Continuous Random Variables

- Probability Density Functions
  - Understand the properties of probability density functions
  - Compute probability using a continuous probability density function
• Cumulative Distribution Functions
  • Understand the properties of a cumulative distribution function
  • Find the cumulative distribution function given a probability density function

22.3 Parameters of Continuous Random Variables
• Mean and Median of a Probability Density Function
  • Find the mean of a continuous probability density function
  • Find the median of a continuous probability density function
• Other Parameters of Probability Density Functions
  • Find the variance and standard deviation of a continuous probability density function

22.4 Special Probability Distributions
• Uniform Distribution
  • Compute probability using the uniform distribution
  • Compute the mean, median, and standard deviation of the uniform distribution
• Exponential Distribution
  • Compute probability using the exponential distribution
  • Compute the mean, median, and standard deviation of the exponential distribution
• Normal Distribution
  • Understand the notation and interpret the parameters of a normal distribution
  • Compute z-scores and use them to compare values from different data sets
  • Use a table to find probabilities in a normal distribution