

# Precalculus



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Alta Precalculus was developed to meet the scope and sequence of a typical one-semester precalculus course. To develop the course, Knewton used three main sources of content: Openstax, videos created by a Math Professor we have partnered with, and a team of internal and external Subject Matter Experts. The SMEs come from diverse backgrounds and are all academics in the field of mathematics.

Alta Precalculus has two instructional sequences for every learning objective, giving students multiple opportunities to learn new concepts. Between our OpenStax instructional texts, our videos, and a network of SMEs, we were able to solicit ideas from math instructors and students. Alta Precalculus covers the typical breadth of precalculus topics, and also provides the necessary depth to ensure the course is manageable and engaging for instructors and students alike.

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## Precalculus | Table of Contents

### Chapter 1: Prerequisites

#### 1.1 Algebra Essentials

- Sets and Venn Diagrams
  - Represent a set using a written description and the roster method
  - Identify subsets, universal sets, and empty sets
  - Illustrate two sets using a Venn diagram and set notation
- Set Operations
  - Determine the complement of a set using Venn diagrams or set notation
  - Determine the intersection of two sets using Venn diagrams and set notation
  - Determine the union of two sets using Venn diagrams or set notation
  - Perform operations on sets
- Properties of Real Numbers and Order of Operations
  - Distinguish between natural numbers, whole numbers, and integers
  - Distinguish between rational and irrational numbers
  - Perform calculations using order of operations
  - Use the inverse and identity properties of real numbers
  - Use the commutative, associative, and distributive properties of real numbers
- Evaluate and Simplify Algebraic Expressions
  - Evaluate algebraic expressions with a single variable
  - Evaluate algebraic expressions with two variables
  - Identify constants and variables
  - Use a formula
  - Simplify algebraic expressions
- Evaluate and Simplify Absolute Value Expressions
  - Understand absolute value and evaluate absolute value expressions with the order of operations
  - Evaluate algebraic expressions involving absolute value

#### 1.2 Exponents and Scientific Notation

- Product, Quotient, and Power Properties of Exponents
    - Understand exponent notation
    - Use the product rule of exponents
    - Use the quotient rule of exponents
    - Use the power rule of exponents
  - Negative Exponents and Simplifying Exponential Expressions
    - Use the negative and zero exponent rule
    - Find the power of a product
    - Find the power of a quotient
    - Simplify exponential expressions
  - Scientific Notation
    - Convert between standard and scientific notation
    - Multiply and divide numbers in scientific notation
-

### 1.3 Radicals and Rational Exponents

- Simplify Radicals
  - Evaluate square roots
  - Use the product rule to simplify square roots
  - Use the quotient rule to simplify square roots
- Operations with Radicals
  - Add and subtract square roots
  - Rationalize denominators with a monomial denominator
  - Rationalize denominators using the conjugate
- Rational Exponents and Higher Order Radicals
  - Simplify  $n$ th roots
  - Operations with  $n$ th roots
  - Switch between radical and rational exponent form
  - Evaluate expressions with rational exponents

### 1.4 Polynomials

- Properties of Polynomials
  - Identify the degree and leading coefficient of a polynomial
  - Identify monomials, binomials, and trinomials
- Operations on Polynomials
  - Add and subtract polynomials
  - Multiply binomials together
  - Multiply polynomials together
  - Perform operations with polynomials of several variables

### 1.5 Factoring Polynomials

- Factor Quadratics
  - Factor the greatest common factor of a polynomial
  - Factor a trinomial
  - Factor a trinomial by grouping
- Factor Quadratics with Special Products
  - Factor a perfect square trinomial
  - Factor a difference of squares
- Factor Cubics
  - Factor a cubic by grouping
  - Factor the sum and difference of cubes
- Factor Expressions with Fractional or Negative Exponents
  - Factor expressions using fractional or negative exponents
  - Factor expressions using greatest common factor and other technique

### 1.6 Rational Expressions

- Multiply and Divide Rational Expressions
    - Simplify rational expressions
    - Multiply rational expressions
    - Divide rational expressions
-

- Add and Subtract Rational Expressions and Simplify Complex Rational Expressions
  - Add and subtract rational expressions
  - Simplify complex rational expressions

## Chapter 2: Equations and Inequalities

### 2.1 The Rectangular Coordinate Systems and Graphs

- Cartesian Coordinates and Distances
  - Plot ordered pairs in a Cartesian coordinate system (\*21)
  - Graph equations by plotting points (\*21)
  - Use the distance formula, given two points
  - Use the midpoint formula (\*10)

### 2.2 Linear and Rational Equations in One Variable

- Solve Linear Equations in One Variable
  - Identify identity, conditional, and inconsistent equations
  - Solve equations in one variable algebraically, variable just on one side
  - Solve equations in one variable algebraically, variable on both sides
- Solve Rational Equations
  - Solve a rational equation, monomials in denominator
  - Solve a rational equation, binomials in denominator
  - Solve a rational equation, requires factoring to find least common denominator
- Identify Slopes and Intercepts
  - Find the slope of a line given two points
  - Understand the relationship between the slope and y-intercept of a line and its equation
  - Find x -intercepts and y -intercepts
- Find Linear Equations
  - Find equation of a line, in slope-intercept form, given slope and one point (point-slope formula)
  - Find equation, in slope-intercept form, of a line passing through two given points
  - Given slope and intercept, find the equation of a line and write it in standard form
  - Find the equation of vertical and horizontal lines
- Parallel and Perpendicular Lines
  - Given the equations of two lines, determine whether their graphs are parallel or perpendicular
  - Write the equation of a line parallel to a given line
  - Write the equation of a line perpendicular to a given line

### 2.3 Models and Applications

- Word Problems with Linear Equations
    - Set up a linear equation to solve a real-world application
    - Translate verbal expressions into mathematical expressions
    - Use a formula to solve a real-world application
  - Problem Solving
    - Solve simple interest applications
    - Solve a formula for a specified variable
-

- Use a formula to solve a geometric application
- Use the Pythagorean theorem

#### 2.4 Complex Numbers

- Basics of Complex Numbers
  - Express the square root of a negative number as a multiple of  $i$
  - Simplify powers of  $i$
- Operations on Complex Numbers
  - Add and subtract complex numbers
  - Multiply a complex number by a real number
  - Multiply two complex numbers
  - Divide two complex numbers

#### 2.5 Quadratic Equations

- Solve Quadratic Equations by Factoring
  - Solve quadratic equations by factoring, leading coefficient 1
  - Solve quadratic equations by factoring, leading coefficient  $> 1$
- Complete the Square
  - Solve quadratic equations by the square root property
  - Solve quadratic equations by completing the square
- Quadratic Formula
  - Use the discriminant to classify the solutions of a quadratic equation
  - Solve quadratic equations by using the quadratic formula

#### 2.6 Other Types of Equations

- Solve Higher Order Equations with Factoring
  - Solve equations by factoring out the greatest common factor
  - Solve equations by factoring with grouping
- Solve Equations Quadratic in Form by Factoring
  - Solve fourth-degree equation in quadratic form
  - Solve quadratic with binomial
- Solve Radical Equations
  - Solve radical equations with a single radical
  - Solve radical equations with two radicals
- Solve Other Types of Equations
  - Solve equations using reciprocal exponents
  - Solve equations involving rational exponents by factoring out the greatest common factor
  - Solve rational equation which leads to a quadratic

#### 2.7 Linear Inequalities and Absolute Value Inequalities

- Interval Notation and Inequalities
    - Use interval notation
    - Use properties of inequalities
    - Solve simple inequalities in one variable algebraically
    - Solve compound inequalities in one variable algebraically
  - Absolute Value Equations and Inequalities
    - Solve absolute value equations
    - Solve absolute value inequalities
-

- Applications with Linear Inequalities
  - Translate words to an inequality and solve applications with linear inequalities
  - Solve applications with compound inequalities
  - Solve applications with absolute value

## 2.8 Inequalities Requiring Factoring

- Rational and Quadratic Inequalities
  - Solve quadratic inequalities in one variable, graph the solution set, and express the solution set using interval notation
  - Solve inequalities that involve rational expressions, graph the solution sets, and express the solution set using interval notation

## Chapter 3: Functions

### 3.1 Functions and Function Notation

- Relations and Functions
  - Identify domain and range from a set of ordered pairs
  - Determine whether a relation represents a function
  - Use the vertical line test to identify functions
- One-to-One Functions
  - Determine whether a function is one-to-one
  - Use the horizontal line test to identify one-to-one functions
- Function Notation
  - Evaluate a function using function notation
  - Solve a function using function notation
  - Evaluate or solve a function from a table
  - Evaluate or solve a function from a graph

### 3.2 Absolute Value Functions and Other Toolkit Functions

- Graph Absolute Value Functions
  - Graph an absolute value function (\*20)
- Toolkit Functions
  - Identify graphs of toolkit functions

### 3.3 Domain and Range

- Domain and Range of Functions
  - Find the domain of a function defined by an equation
  - Find the domain and range of a function defined by a graph
- Piecewise Functions
  - Graph piecewise-defined functions
  - Evaluate piecewise-defined functions

### 3.4 Rates of Change and Behavior of Graphs

- Graphical Properties of Functions
    - Find the average rate of change of a function
    - Use a graph to determine intervals of increase and decrease and local extrema
    - Use a graph to locate the absolute maximum and absolute minimum
  - Difference Quotients
    - Determine the difference quotient
-

### 3.5 Composition of Functions

- Combinations of Functions
  - Combine functions using algebraic operations
  - Create a new function by composition of functions
- Evaluate Composite Functions
  - Evaluate composite functions given a table of values
  - Evaluate composite functions given the graph of functions
  - Evaluate composite functions given explicit functions
- Properties of Composite Functions
  - Find the domain of a composite function
  - Decompose a composite function into its component functions

### 3.6 Function Graphs and Transformations

- Transformations of Functions
  - Graph functions using vertical and horizontal shifts (\*20)
  - Graph functions using reflections about the x-axis and the y-axis (\*20)
  - Graph functions using compressions and stretches (\*20)
  - Combine transformations (\*20)
- Even and Odd Functions
  - Determine whether a function is even, odd, or neither from its graph
  - Determine whether a function is even, odd, or neither given algebraically

### 3.7 Inverse Functions

- Inverse Function Values
  - Verify inverse function ordered pairs
  - Given graph of a function, find value of inverse function (\*10)
  - Given table of values of a function, find value of inverse function
- Find Inverse Functions
  - Verify inverse function pairs algebraically
  - Determine the domain and range of an inverse function, and restrict the domain of a function to make it one-to-one
  - Given function, find the inverse function
  - Use the graph of a one-to-one function to graph its inverse function on the same axes

### 3.8 Circles

- Graphs of Circles
  - Given the equation of a circle not in standard form, determine the standard form by completing the square
  - Determine the center and radius of a circle from the standard equation of a circle and sketch its graph (\*20)

## Chapter 4: Linear Functions and Modeling

### 4.1 Linear Functions

- Interpretations of Linear Functions
    - Represent a linear function in table form
    - Determine whether a linear function is increasing, decreasing, or constant
    - Interpret slope as a rate of change
-



- Represent a real-world application as a linear function
- Graph linear functions

#### 4.2 Modeling with Linear Functions

- Application of Linear Functions
  - Build linear models from verbal descriptions, given a y-intercept
  - Build linear models from verbal descriptions, given inputs and outputs
  - Use a diagram to build a model
  - Model a set of data with a linear function

#### 4.3 Fitting Linear Models to Data

- Scatter Diagrams and Lines of Best Fit
  - Draw and interpret scatter diagrams (\*38)
  - Distinguish between linear and nonlinear relations
- Linear Regressions
  - Find the line of best fit using a graphing utility
  - Use the line of best fit to make predictions

### Chapter 5: Polynomial and Rational Functions

#### 5.1 Quadratic Functions

- Characteristics of Parabolas
  - Determine axis of symmetry and vertex of parabolas from a graph
  - Determine x- and y-intercepts of parabolas from a graph
  - Find the direction a parabola opens and its axis of symmetry and vertex from the general form of its equation
  - Identify the axis of symmetry and vertex of a parabola from its equation in standard form
- Graphs of Quadratic Functions
  - Write the equation of a quadratic function given vertex and a point on a graph
  - Write the equation of a quadratic function given intercepts on a graph
  - Write the equation of a quadratic function in standard form given the equation in general form
- Applications of Quadratic Functions
  - Find the domain and range of a quadratic function
  - Determine the maximum and minimum values of quadratic functions
  - Find the x- and y-intercepts of a quadratic function
  - Use a quadratic function to model projectile motion
- Quadratic Regressions
  - Find the parabola of best fit using a graphing utility
  - Use the parabola of best fit to make predictions

#### 5.2 Graphs of Polynomial and Power Functions

- End Behavior of Polynomial Functions
    - Identify power functions and polynomial functions
    - Identify if a graph is a polynomial function
    - Determine end behavior
-

- Local Behavior of Polynomial Functions
  - Identify intercepts of polynomial functions in factored form
  - Understand the relationship between degree, turning points, and x-intercepts
  - Understand the intermediate value theorem
  - Use factoring to find zeros of polynomial functions
  - Identify zeros and their multiplicities from an equation or a graph
- Write and Graph Polynomial Functions
  - Draw conclusions about a polynomial function from a graph
  - Graph polynomial functions (\*20)
  - Write a formula for a polynomial function from a graph
  - Determine equation of a polynomial given key information

### 5.3 Dividing Polynomials

- Long Division of Polynomials
  - Use long division to divide polynomials
  - Use polynomial division to solve application problems
- Synthetic Division and Remainder Theorem
  - Use synthetic division to divide polynomials
  - Evaluate a polynomial using the remainder theorem

### 5.4 Zeros of Polynomial Functions

- Rational Zeros of Polynomial Functions
  - Use the factor theorem to solve a polynomial equation
  - Use the rational zero theorem to find rational zeros
  - Solve real-world applications of polynomial equations
- Complex Zeros of Polynomial Functions
  - Find zeros of polynomial functions with complex zeros
  - Use the linear factorization theorem to find polynomials with given zeros
  - Use Descartes' rule of signs

### 5.5 Rational Functions

- Asymptotic Behavior of Rational Functions
  - Use arrow notation to describe local behavior and end behavior of rational functions
  - Identify vertical asymptotes and removable discontinuities of rational functions
  - Identify horizontal and slant asymptotes of rational functions
- Graphs and Applications of Rational Functions
  - Solve applied problems involving rational functions
  - Find the intercepts of a rational function
  - Graph rational functions
  - Find the equation of a rational function from a graph

### 5.6 Inverses and Radical Functions

- Inverses of Polynomial Functions
    - Find the inverse of an invertible polynomial function
    - Restrict the domain to find the inverse of a polynomial function
    - Solve an application with the inverse of a function
-

- Inverses of Radical and Rational Functions
  - Find the inverse of a radical function
  - Find the domain of a radical function composed with a rational function
  - Find the inverse of a rational function

#### 5.7 Modeling Using Variation

- Direct and Inverse Variation
  - Solve direct variation problems
  - Solve inverse variation problems
  - Solve problems involving joint variation

### Chapter 6: Exponential and Logarithmic Functions

#### 6.1 Exponential Functions

- Evaluate and Write Exponential Functions
  - Identify exponential functions
  - Evaluate exponential functions
  - Find the equation of an exponential function given the initial value and a point
  - Find the equation of an exponential function when the initial value is not known
- Applications of Exponential Functions and Base  $e$ 
  - Find the equation of an exponential function in a word problem context
  - Calculate compound interest
  - Evaluate exponential functions with base  $e$
  - Calculate continuous growth and decay

#### 6.2 Graphs of Exponential Functions

- Exponential Function Graphs
  - Graph exponential functions (\*10)
  - Graph exponential functions using transformations (\*21)
  - Find the equation of an exponential function given a graph
  - Write an exponential function from a description

#### 6.3 Logarithmic Functions

- Relate Logarithms and Exponents
  - Convert from logarithmic to exponential form
  - Convert from exponential to logarithmic form
- Evaluate Logarithmic Expressions
  - Evaluate logarithms with positive integer solutions
  - Evaluate logarithms with negative integer solutions
  - Use common logarithms
  - Use natural logarithms

#### 6.4 Graphs of Logarithmic Functions

- Logarithmic Function Graphs
    - Identify the domain of a logarithmic function
    - Graph logarithmic functions (\*13)
    - Graph transformations of logarithmic functions
    - Write a logarithmic function from a description
-

## 6.5 Logarithmic Properties

- Basic Properties of Logarithms
  - Understand the basic properties of logarithms
  - Use the product rule for logarithms
  - Use the quotient rule for logarithms
  - Use the power rule for logarithms
- Rewrite Logarithmic Expressions Using Properties
  - Expand logarithmic expressions
  - Condense logarithmic expressions
  - Use the change-of-base formula for logarithms

## 6.6 Exponential and Logarithmic Equations

- Solve Exponential Equations
  - Use like bases to solve exponential equations
  - Rewrite equations so all powers have the same base and solve exponential equations
  - Use logarithms to solve exponential equations
  - Solve an equation with a base  $e$  using natural logarithms
- Solve Logarithmic Equations
  - Use the definition of a logarithm to solve logarithmic equations
  - Use logarithm properties and the definition of the logarithm to solve logarithmic equations
  - Use the one-to-one property of logarithms to solve logarithmic equations

## 6.7 Exponential and Logarithmic Models

- Applications of Exponential and Logarithmic Functions
  - Model exponential growth
  - Model exponential decay
  - Applied logarithmic models
  - Choose an appropriate model for data
  - Express an exponential model in base  $e$

## 6.8 Fitting Exponential Models to Data

- Exponential and Logarithmic Regressions
  - Build an exponential model from data using a graphing utility
  - Build a logarithmic model from data using a graphing utility
- Logistic Growth Models
  - Use logistic growth models
  - Build a logistic model from data using a graphing utility

## Chapter 7: Right Triangle Trigonometry and the Unit Circle

### 7.1 Angles as Rotations and Arc Length

- Angles as Rotations and Radian Measures
    - Identify the measure of positive and negative angles in standard position and the quadrant of the terminal side
    - Convert between degree and radian measure of an angle
    - Understand when two angles are coterminal
-

- Arc Length and Area of a Sector
  - Find the length of an arc
  - Find the area of a sector
  - Understand the relationship between linear and angular speed

## 7.2 Right Triangle Trigonometry

- The Six Trigonometric Ratios
  - Use right triangles to evaluate sine, cosine, and tangent functions
  - Evaluate reciprocal trig functions using right triangles or a sine, cosine, or tangent function
  - Evaluate trigonometric functions of angles not in standard position
- Use Right Triangle Trigonometry in Solving Problems
  - Find missing side lengths using trig ratios
  - Use right triangle trigonometry to solve applied problems

## 7.3 The Unit Circle

- Sine and Cosine Values in the First Quadrant
  - Understand sine and cosine values on the unit circle
  - Find exact sine and cosine values for angles in the first quadrant of the unit circle
- Sine and Cosine Values with Reference Angles
  - Find the reference angle for a given angle
  - Use reference angles to evaluate sine and cosine functions
  - Use reference angles to find coordinates on the unit circle
  - Evaluate sine and cosine functions with a calculator

## 7.4 The Other Trigonometric Functions

- The Other Trigonometric Ratios on the Unit Circle
  - Find the secant, cosecant, tangent, and cotangent values for angles in the first quadrant of the unit circle
  - Use reference angles to evaluate secant, cosecant, tangent, and cotangent functions
  - Evaluate trigonometric functions with a calculator
- Use Given Trigonometric Ratios to Find Other Ratios
  - Understand the relationship between the quadrant in which an angle falls and the signs of the trig functions of that angle
  - Use the pythagorean identity
  - Find the values of all trigonometric functions given coordinates on a unit circle
  - Find the values of all trigonometric functions given the value of one trigonometric function

## Chapter 8: Periodic Functions

### 8.1 Sine and Cosine Graphs

- Characteristics of Sine and Cosine Graphs
    - Graph the sine function and understand its properties
    - Graph the cosine function and understand its properties
  - Transformations of Sine and Cosine Graphs
    - Determine the period and amplitude of a sinusoidal function
    - Determine the phase shift and vertical shift of a sinusoidal function
-

- Graph Sine and Cosine Functions
  - Find the equation of a sinusoidal function given a graph
  - Find the graph of a sinusoidal function given equation (\*20)
  - Use sinusoidal functions to solve real-world applications

## 8.2 Graphs of Other Trigonometric Functions

- Characteristics of Tangent and Cotangent Graphs
  - Graph tangent functions
  - Graph cotangent functions
- Transformations of Tangent and Cotangent Functions
  - Graph tangent or cotangent functions over different periods (\*21)
  - Graph transformations of tangent and cotangent functions
- Characteristics of Secant and Cosecant Graphs
  - Graph cosecant functions
  - Graph secant functions
- Graph Secant and Cosecant Functions
  - Graph transformations of cosecant functions (\*3)
  - Graph transformations of secant functions
  - Find the equation of a cosecant function from a graph
  - Find the equation of a secant function from a graph

## 8.3 Inverse Trigonometric Functions

- Introduction to Inverse Trigonometric Functions
  - Understand inverse sine, cosine, and tangent functions
  - Understand inverse secant, cosecant, and cotangent functions
  - Use a calculator to evaluate inverse trigonometric functions
- Solve Triangles with Inverse Trigonometric Functions
  - Find an angle given two sides of a right triangle
  - Solve right triangle problems
- Compose Functions with Inverse Trigonometric Functions
  - Evaluate composite functions with inverse trigonometric functions in the form  $f(f^{-1}(x))$  or  $f(g^{-1}(x))$
  - Evaluate composite functions with inverse trigonometric functions in the form  $f^{-1}(f(x))$  or  $f^{-1}(g(x))$

## Chapter 9: Trigonometric Identities and Equations

### 9.1 Fundamental Trigonometric Identities

- Simplify Expressions with Basic Trigonometric Identities
    - Understand quotient and reciprocal identities
    - Use even and odd identities in simplifying trigonometric expressions
  - Use Pythagorean and Cofunction Identities
    - Understand all forms of the pythagorean identity
    - Use the cofunction identities
  - Verify Trigonometric Identities
    - Use all identities to simplify trigonometric expressions
    - Use algebraic techniques to simplify trigonometric expressions
-

## 9.2 Sum and Difference Identities

- Sum and Difference Formulas
  - Use the sum and difference formula for cosine
  - Use the sum and difference formula for sine
  - Use the sum and difference formula for tangent
  - Use the sum and difference formulas to simplify trigonometric expressions

## 9.3 Double-Angle, Half-Angle, and Reduction Formulas

- Double-Angle Formulas
  - Use double-angle formulas to find values of trigonometric functions
  - Use double-angle formulas to simplify trigonometric expressions
- Half-Angle and Power-Reduction Formulas
  - Use reduction formulas to simplify an expression
  - Use half-angle formulas to find values of trigonometric functions

## 9.4 Sum-to-Product and Product-to-Sum Formulas

- Sum-to-Product and Product-to-Sum Formulas
  - Express the product of trigonometric functions as a sum
  - Express sums of trigonometric functions as a product

## 9.5 Solving Trigonometric Equations

- Trigonometric Equations in Sine and Cosine
  - Solve linear trigonometric equations in sine
  - Solve linear trigonometric equations in cosine
- Trigonometric Equations Involving a Single Trigonometric Function
  - Solve linear trigonometric equations in secant and cosecant
  - Solve linear trigonometric equations in tangent and cotangent
  - Solve trigonometric equations using a calculator
- Trigonometric Equations in Quadratic Form or Requiring Factoring
  - Solve equations with a single trigonometric function
  - Solve factorable trigonometric equations in quadratic form
  - Solve trigonometric equations in quadratic form requiring the quadratic formula
- Trigonometric Equations Requiring Identities or Multiple Angles
  - Solve trigonometric equations using fundamental identities
  - Solve trigonometric equations with multiple angles
  - Solve trigonometric equations using cofunction identities

## Chapter 10: Further Applications of Trigonometry

### 10.1 Non-right Triangles - Law of Sines

- Law of Sines
    - Use the law of sines to solve ASA or AAS triangles
    - Use the law of sines to solve SSA triangles
    - Solve applied problems with the law of sines
-

## 10.2 Non-right Triangles - Law of Cosines and Area of Oblique Triangles

- Law of Cosines
  - Use the law of cosines to solve SAS triangles
  - Use the law of cosines to solve SSS triangles
  - Solve applied problems with the law of cosines
- Area of Oblique Triangles
  - Find the area of an oblique triangle using the sine function
  - Use Heron's formula to find the area of a triangle

## 10.3 Polar Coordinates

- Convert Coordinates Between Rectangular and Polar Forms
  - Plot points using polar coordinates
  - Convert from polar coordinates to rectangular coordinates
  - Convert from rectangular coordinates to polar coordinates
- Convert Equations Between Rectangular and Polar Forms
  - Write a cartesian equation in polar form
  - Write a polar equation in cartesian form

## 10.4 Graphs in Polar Coordinates

- Introductions to Graphing Polar Equations
  - Test a polar equation for symmetry
  - Find zeros and maximum values for a polar equation and graph polar equations by plotting points
  - Graph a circle or a line from a polar equation
- Graph Classic Polar Curves
  - Graph a cardioid from a polar equation
  - Graph a limaçon from a polar equation
  - Graph a lemniscate from a polar equation
  - Graph a rose curve from a polar equation

## 10.5 Polar Form of Complex Numbers

- Write Complex Numbers in Polar Form
  - Plot complex numbers
  - Find the absolute value of a complex number
  - Write complex numbers in polar form
  - Convert a complex number from polar to rectangular form
- Product and Quotient of Complex Numbers in Polar Form
  - Find the product of complex numbers in polar form
  - Find the quotient of complex numbers in polar form
- Powers and Roots of Complex Numbers in Polar Form
  - Find powers of complex numbers in polar form
  - Find roots of complex numbers in polar form

## 10.6 Parametric Equations

- Write Parametric Equations
    - Parameterize a curve
    - Find the parametric equations for a line segment given an orientation
-



- Eliminate the Parameter
  - Eliminate the parameter in linear equations
  - Eliminate the parameter in polynomial and radical equations
  - Eliminate the parameter in exponential and logarithmic equations
  - Eliminate the parameter in trigonometric parametric equations

#### 10.7 Graphs with Parametric Equations

- Graph Parametric Equations
  - Graph parametric equations by plotting points
  - Graph trigonometric parametric equations by plotting points
  - Use parametric equations in applications

#### 10.8 Vectors

- Properties of Vectors
  - Understand properties of vectors and find the position vector
  - Find magnitude and direction of a vector
- Vector Additions and Scalar Multiplication
  - Add or subtract vectors (\*11)
  - Multiply a vector by a scalar
  - Use vector addition and scalar multiplication to find a new vector
- The Unit Vector
  - Write a vector in terms of  $i$  and  $j$
  - Find the unit vector
  - Perform operations on vectors in terms of  $i$  and  $j$
  - Write a vector in terms of magnitude and direction (\*10)
- The Dot Product and Vector Applications
  - Find the dot product of two vectors
  - Find the angle between two vectors
  - Use vectors in applications

### **Chapter 11: Systems of Equations and Inequalities**

#### 11.1 Systems of Linear Equations in Two Variables

- Graphing Systems of Linear Equations
    - Determine whether an ordered pair is a solution to a system of equations
    - Solve systems of equations in two variables by graphing
  - Solving Systems of Linear Equations
    - Solve systems of equations in two variables by substitution
    - Solve systems of equations in two variables by addition
    - Identify inconsistent and dependent systems of equations containing two variables, and express the solution of dependent equations
  - Applications of Systems of Linear Equations
    - Use systems of equations to investigate profits
    - Write and solve a system of equations in two variables from a word problem
  - Linear Inequalities in Two Variables
    - Solve a linear inequality in two variables by graphing (\*20)
    - Solve a linear system of inequalities by graphing (\*20)
-

## 11.2 Systems of Linear Equations in Three Variables

- Systems of Linear Equations in Three Variables
  - Determine whether an ordered triple is a solution to a system
  - Solve systems of three equations in three variables
  - Identify inconsistent and dependent systems of equations containing three variables, and express the solution of a system of dependent equations

## 11.3 Systems of Nonlinear Equations in Two Variables

- Systems of Two Nonlinear Equations
  - Solve a system of nonlinear equations representing a parabola and a line
  - Solve a system of nonlinear equations representing a circle and a line
  - Solve a system of nonlinear equations in two variables using elimination
- Graphing Nonlinear Inequalities and Systems of Inequalities
  - Graph a nonlinear inequality (\*10)
  - Graph a system of nonlinear inequalities (\*15)

## 11.4 Partial Fractions

- Partial Fraction Decomposition with Linear Factors
  - Decompose a rational expression where the denominator has only nonrepeated linear factors
  - Decompose a rational expression where the denominator has repeated linear factors
- Partial Fraction Decomposition with Quadratic Factors
  - Decompose a rational expression where the denominator has a nonrepeated irreducible quadratic factor
  - Decompose a rational expression where the denominator has a repeated irreducible quadratic factor

## 11.5 Matrices and Matrix Operations

- Introduction to Matrices
  - Determine the order of a matrix and describe elements within a matrix
  - Add or subtract matrices
- Matrix Multiplication
  - Multiply a matrix by a scalar
  - Find the sum or difference of scalar multiples
  - Multiply two matrices

## 11.6 Augmented Matrices and Gaussian Elimination

- Solving Systems with Gaussian Eliminations
  - Convert between a system of equations and its corresponding augmented matrix
  - Use row operations to solve a system of linear equations in two variables
  - Use row operations to solve a system of linear equations in three variables
  - Use matrices to solve applications of systems of linear equations
- Solving Systems with Gauss-Jordan Elimination
  - Use Gauss-Jordan elimination to solve a system of linear equations

## 11.7 Determinants of Matrices and the Inverse Matrix

- Finding Determinants of Matrices
    - Find the determinant of a 2x2 matrix
    - Find the determinant of a 3x3 matrix
-

- Inverse and Identity Matrices
  - Understand the identity matrix and how it relates to the inverse matrix
  - Determine if a matrix is invertible using the determinant
  - Find the inverse of a 2x2 matrix
  - Find the inverse of a 3x3 matrix
- Solving Systems with Inverses
  - Solve a system of linear equations using the inverse of a 2x2 matrix
  - Solve a system of linear equations using the inverse of a 3x3 matrix

#### 11.8 Cramer's Rule

- Solving Systems with Cramer's Rule
  - Use Cramer's rule to solve a system of two equations in two variables
  - Use Cramer's rule to solve a system of three equations in three variables
  - Use Cramer's rule to solve inconsistent or dependent systems

#### 11.9 Linear Programming

- Using Linear Programming
  - Graph a feasible region given a set of constraints
  - Find the maximum value of an objective function given constraints by graphing
  - Solve application problems using linear programming

### Chapter 12: Conic Sections

#### 12.1 Ellipses

- Ellipses Centered at the Origin
  - Identify key points and axes of ellipses from a graph
  - Identify key points and axes of ellipses from an equation
  - Write the equation in standard form of an ellipse centered at the origin
  - Graph an ellipse centered at the origin from an equation in standard form (\*21)
- Ellipses Not Centered at the Origin
  - Identify key points and axes of ellipses not centered at the origin
  - Write the equation in standard form of an ellipse not centered at the origin
  - Graph an ellipse not centered at the origin (\*21)
- Ellipses Not in Standard Form and Applications of Ellipses
  - Convert an equation of an ellipse into standard form
  - Graph an ellipse where the equation is not given in standard form (\*11)
  - Use ellipses in applications

#### 12.2 Hyperbolas

- Hyperbolas Centered at the Origin
    - Locate the vertices and foci of a hyperbola from a graph
    - Identify vertices, foci, and asymptotes of a hyperbola from an equation
    - Write the equation of a hyperbola centered at the origin in standard form
    - Graph a hyperbola centered at the origin from an equation in standard form (\*20)
  - Hyperbolas Not Centered at the Origin
    - Identify vertices, foci, and asymptotes of a hyperbola not centered at the origin
    - Write the equation of a hyperbola not centered at the origin
    - Graph a hyperbola not centered at the origin from an equation in standard form (\*21)
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- Hyperbolas Not in Standard Form and Applications of Hyperbolas
  - Convert an equation of a hyperbola into standard form
  - Graph a hyperbola from an equation given in general form (\*11)
  - Use hyperbolas in applications

### 12.3 Parabolas

- Parabolas Centered at the Origin
  - Identify key components of a parabola from a graph
  - Identify key components of a parabola from an equation
  - Graph a parabola centered at the origin
  - Write the equation of a parabola centered at the origin in standard form
- Parabolas Not Centered at the Origin
  - Identify key components of a parabola not centered at the origin
  - Graph a parabola not centered at the origin (\*31)
  - Write the equation of a parabola not centered at the origin in standard form
- Parabolas Not in Standard Form and Applications of Parabolas
  - Convert an equation of a parabola into standard form
  - Graph a parabola from an equation given in general form (\*20)
  - Use parabolas in applications

### 12.4 Rotation of Axes

- Conics in General Form and Rotation of Conics
  - Identify a conic from its general form
  - Find the location of a point after a rotation of axes
  - Find a new representation of an equation after rotating through a given angle

### 12.5 Conic Sections in Polar Coordinates

- Conic Sections in Polar Coordinates
  - Identify the type of conic from its polar equation
  - Find the polar equation of a conic given its focus, eccentricity, and directrix

## Chapter 13: Sequences, Probability, and Counting Theory

### 13.1 Sequences

- Introduction to Sequences
  - Write the terms of a sequence defined by an explicit formula
  - Write the terms of a sequence defined by a piecewise explicit formula
- Recursive Sequences
  - Write the terms of a sequence defined by a recursive formula
  - Write the terms of a sequence defined by a recursive formula with more than one initial term

### 13.2 Arithmetic Sequences

- Arithmetic Sequences
    - Find the common difference of an arithmetic sequence
    - Write terms of an arithmetic sequence
    - Write a recursive formula for an arithmetic sequence
    - Write an explicit formula for an arithmetic sequence
-

- Applications of Arithmetic Sequences
  - Find specific terms of an arithmetic sequence given other terms
  - Solve application problems with arithmetic sequences

### 13.3 Geometric Sequences

- Geometric Sequences
  - Find the common ratio of a geometric sequence
  - Write terms of a geometric sequence
  - Write a recursive formula for a geometric sequence
  - Write an explicit formula for a geometric sequence
- Applications of Geometric Sequences
  - Write an explicit formula for the  $n$ th term of a sequence
  - Solve application problems with geometric sequences
  - Solve geometric sequence problems

### 13.4 Series

- Summation Notation and Arithmetic Series
  - Evaluate expressions using summation notation
  - Find the sum of a finite arithmetic series
- Finite and Infinite Geometric Series
  - Find the sum of a finite geometric series
  - Determine if the sum of an infinite series is defined
  - Find the sum of an infinite geometric series
- Applications of Series
  - Solve application problems with arithmetic series
  - Solve application problems with geometric series
  - Find the equivalent fraction for a repeating decimal
  - Solve an annuity problem

### 13.5 Counting Theory

- The Addition and Multiplication Principles
  - Solve counting problems using the addition principle
  - Solve counting problems using the multiplication principle
  - Evaluate an expression with factorials
- Permutations
  - Find the number of permutations of  $n$  distinct objects using the multiplication principle
  - Find the number of permutations of  $n$  distinct objects using a formula
  - Find the number of permutations of  $n$  non-distinct objects
- Combinations
  - Find the number of combinations using the formula
  - Find the number of subsets of a set

### 13.6 Binomial Theorem

- Binomial Expansion
    - Find a binomial coefficient
    - Expand a binomial using the binomial theorem
    - Use the binomial theorem to find a single term
-

### 13.7 Probability

- Basic Probability
  - Compute the probability of equally likely outcomes
  - Compute the probability of the union of two events
  - Use the complement rule to compute probabilities
  - Compute probability using counting theory
- Binomial Probability
  - Identify a binomial experiment
  - Determine the binomial probability of success in an experiment performed multiple times

## Chapter 14: Introduction to Calculus

### 14.1 Finding Limits Using Numerical and Graphical Approaches

- Limits From a Graph or Table
  - Understand the limit of a function and evaluate a limit from a table
  - Evaluate limits graphically
  - Understand the properties of limits

### 14.2 Finding Limits Analytically

- Limits Analytically for Continuous and Piecewise Functions
    - Evaluate two-sided limits analytically for continuous functions
    - Evaluate limits analytically for piecewise functions
    - Evaluate limits analytically for absolute value functions
  - Limits Analytically for Functions with Removable Discontinuities
    - Evaluate two-sided limits analytically for rational functions with removable discontinuities by factoring
    - Evaluate two-sided limits analytically for rational functions with removable discontinuities through expansion
    - Evaluate two-sided limits analytically for complex fractions with removable discontinuities
    - Evaluate two-sided limits analytically for rational functions that contain radicals with removable discontinuities
  - Limits Analytically for Trigonometric Functions
    - Evaluate trigonometric limits using direct substitution
    - Evaluate trigonometric limits using special limits
    - Evaluate trigonometric limits using identities
  - Infinite Limits
    - Evaluate limits analytically for functions with essential discontinuities
    - Evaluate limits analytically for trigonometric functions with essential discontinuities
  - Limits at Infinity
    - Evaluate limits of polynomial functions at infinity
    - Evaluate limits of rational functions at infinity
    - Evaluate limits of trigonometric functions at infinity
    - Evaluate limits of radical and exponential functions at infinity
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### 14.3 Continuity

- Continuity and the Intermediate Value Theorem
  - Understand the definition of continuity
  - Distinguish between types of discontinuity
  - Understand and apply the intermediate value theorem
- Continuity of Piecewise Functions
  - Determine whether a piecewise function is continuous
  - Determine the value that makes a piecewise function continuous

### 14.4 Derivatives

- The Average Rate of Change
  - Find the average rate of change given a function
  - Find the average rate of change given a function and variable intervals
- Derivatives Using the Limit Definition
  - Find the derivative of a function at a point using limits
  - Use the limit definition to find the derivative of a polynomial function
  - Use the limit definition to find the derivative of a rational function
  - Use the limit definition to find the derivative of a function with a radical
- Instantaneous Rates of Change
  - Determine the sign of the slope of a line tangent to a function at a given point
  - Estimate the derivative at a point on the graph of a function
  - Use instantaneous rates of change in applications
- Differentiability
  - Determine where a function is differentiable from a graph

### 14.5 Formal Limits

- Precise Definition of a Limit
    - Understand the notation in finding the formal definition of a limit
    - Determine a delta for an arbitrary epsilon given a quadratic function
    - Determine a delta for an arbitrary epsilon given a linear function
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