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Quantitative Reasoning | Table of Contents

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  - Determine the winner of an election using the Borda count method
- 17.1.2 Comparison Voting Methods
  - Decide the winner of an election using the plurality with elimination method (40)
  - Select the winner of an election using the pairwise comparison method (40)

17.2 Determine a Voting Systems Fairness
- 17.2.1 Criterion of Voting Methods - Majority, Head to Head count, and Monotonicity
  - Determine a voting system's fairness using the monotonicity criterion
  - Determine a voting systems fairness using the majority criterion
  - Determine a voting systems fairness using the head to head count criterion
- 17.2.2 Criterion and Comparison of Voting Methods
  - Determine a voting system's fairness using the irrelevant alternatives criterion
  - Choose the best voting system for a given election

17.3 Apportionment Methods
- 17.3.1 Apportionment Methods by Calculating Quotas
  - Determine the standard quota for a state given its population
  - Calculate the lower and upper quotas for a state given its population
- 17.3.2 Apportionment Methods - Hamilton, Jefferson, Adam and Webster Methods
  - Use Hamilton's method of determining apportionment
  - Use Jefferson's method of determining apportionment (40)
  - Use Adam's method of determining apportionment
  - Use Webster's method of determining apportionment
- 17.3.3 Flaws of the Apportionment Methods
  - Identify where the Alabama paradox occurs (40)
  - Identify where the population paradox occurs
  - Identify where the new states paradox occurs
  - Choose the best apportionment system for a given region

Chapter 18: Basic Math Appendix

18.1 Introduction to Numbers
- 18.1.1 Number Theory
  - Understand and identify prime and composite numbers
  - Find the GCF and LCM of two or more numbers (40)
  - Find the prime factorization of a number

18.2 Operations with Integers
- 18.2.1 The Integers
  - Understand integers and find opposites of numbers (40)
  - Order and compare integers (40)
  - Understand and evaluate absolute value
  - Evaluate an absolute value expression
- 18.2.2 Working with Integers
  - Understand additive inverse (40)
  - Understand distance in terms of absolute value (40)
- 18.2.3 Adding and Subtracting integers
  - Add and subtract integers using order of operations
  - Add and subtract integers (40, 40)
  - Subtract integers
  - Add integers
- 18.2.4 Multiply and Divide integers

- Multiply integers
- Divide integers

18.3 Introduction to Rational Numbers
- 18.3.1 The Rational Numbers
  - Understand fractions and their models
  - Find equivalent fractions (40)
  - Convert between fractions and mixed numbers (40)
- 18.3.2 Converting between Representations
  - Convert a repeating decimal to a fraction
  - Convert fractions to decimals (40)
  - Convert decimals to fractions (40)
  - Convert between percents, decimals, and fractions (40)

18.4 Operations with Fractions
- 18.4.1 Operations on Fractions - Multiplication
  - Multiply fractions
  - Find reciprocals of fractions (40)
  - Divide fractions
- 18.4.2 Combining Fractions
  - Add and subtract fractions with like denominators
  - Add and subtract fractions with unlike denominators
  - Add or subtract fractions with a common denominator
  - Add or subtract fractions with different denominators
  - Add and subtract fractions in applications

18.5 Ratios
- 18.5.1 Ratios and Proportions
  - Solve ratio and unit rate problems (40)
  - Understand and find unit rate
  - Understand ratios (40)
- 18.5.2 Advanced Ratios
  - Solve fractional ratio problems (40)
  - Understand fractional ratios (40)

18.6 Additional Topics in Probability
- 18.6.1 Permutations
  - Find the number of permutations of n distinct objects using the multiplication principle
  - Find the number of permutations of n non-distinct objects
- 18.6.2 Working with subsets and combinations
  - Find the number of subsets of a set
- 18.6.3 Probability of the union of two events
  - Compute the probability of the union of two events
- 18.6.4 Calculate and Interpret the expected value
  - Calculate and interpret the expected value

18.7 Sequences and Series
- 18.7.1 Sequences of Rational Numbers
  - Write the terms of a sequence defined by an explicit formula (40)

- 18.7.2 Write the terms of a sequence defined by a piecewise explicit formula
    - Write the terms of a sequence defined by a piecewise explicit formula
- 18.7.3 Recursive Sequences
    - Write the terms of a sequence defined by a recursive formula
    - Write the terms of a sequence defined by a recursive formula with more than one initial term
- 18.7.4 Arithmetic Sequences
    - Find the common difference of an arithmetic sequence
    - Write terms of an arithmetic sequence (40)
    - Write an explicit formula for an arithmetic sequence (40)
- 18.7.5 Write a recursive formula for an arithmetic sequence
    - Write a recursive formula for an arithmetic sequence
- 18.7.6 Applications of Arithmetic Sequences
    - Find specific terms of an arithmetic sequence given other terms (20)
    - Solve application problems with arithmetic sequences (40)
- 18.7.7 Geometric Sequences
    - Find the common ratio of a geometric sequence (40)
    - Write terms of a geometric sequence
    - Write an explicit formula for a geometric sequence (40)
    - Find the sum of a finite geometric series
- 18.7.8 Write a recursive formula for a geometric sequence
    - Write a recursive formula for a geometric sequence
- 18.7.9 Applications of Geometric Sequences
    - Write an explicit formula for the nth term of a sequence
    - Solve application problems with geometric sequences (40)
    - Solve geometric sequence problems
- 18.7.10 Sums of Series and Notation
    - Evaluate expressions using summation notation
    - Find the sum of a finite arithmetic series

18.8 Slopes of Equations of lines
- 18.8.1 Slopes of Equations of lines
    - Write the equation of a line parallel to a given line
    - Write the equation of a line perpendicular to a given line