

## Single Variable Calculus, Early Transcendentals | Table of Contents

### Chapter 1: A Review of Functions and Graphs

#### 1.1 Review of Functions

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- Determining the Explicit Formula for a Sequence
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- Finding the Limit of a Sequence
  - Use the Squeeze Theorem to find the limit of a sequence
  - Determine if a sequence is bounded (5)
  - Find the limit of a sequence using the Monotone Convergence Theorem

### 9.2 Infinite Series

- Convergent and Harmonic Series
  - Determine whether a series converges or diverges using the sequence of partial sums (5)
  - Evaluate a convergent series using algebraic properties (5)
- Geometric Series
  - Determine if a geometric series converges and if so find its sum
  - Express repeating decimals as fractions using geometric series (5)
  - Evaluate a telescoping series

### 9.3 Convergence Tests for Series

- The Divergence and Integral Tests
  - Determine if a series diverges using the divergence test (5)
  - Determine if a series converges or diverges using the integral test
  - Determine if a  $p$ -series converges or diverges (5)
  - Estimate the value of a convergent series (5)

### 9.4 Comparison Tests

- The Comparison Test
  - Determine if a series converges or diverges using the comparison test (10)
  - Determine if a series converges or diverges using the limit comparison test (10)

### 9.5 Alternating Series

- Alternating Series and the Alternating Series Test
  - Determine if an alternating series converges or diverges using the alternating series test (40)
  - Estimate the remainder of an alternating series (40)
  - Determine whether a series converges absolutely or conditionally (40)

### 9.6 Ratio and Root Tests

- Ratio and Root Tests
    - Determine if a series converges or diverges using the ratio test (5)
    - Determine if a series converges or diverges using the root test (5)
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- When to Use Convergence Tests
  - Choose an appropriate convergence test for a series (5)
  - Determine if a series converges or diverges (5)

## **Chapter 10: Power Series**

### 10.1 Power Series and Functions

- Convergent Power Series
  - Identify a power series (5)
  - Find the interval and radius of convergence for a power series (5)
  - Represent a rational function with a power series (5)
  - Determine which convergence property a power series satisfies

### 10.2 Properties of Power Series

- Combining Power Series
  - Find the interval of convergence of the sum of two power series (5)
  - Find the interval of convergence of the product of a power series and a power (5)
  - Multiply two power series together
- Representing Functions with Power Series
  - Use a power series to solve application problems
  - Find the power series representation of a function using a known power series (5)
  - Find the function represented by a given power series
- Calculus of Power Series
  - Integrate a power series
  - Differentiate a power series (5)
- Differential Equations and Power Series
  - Solve a first order differential equation using a power series
  - Solve Airy's equation using a power series (5)

### 10.3 Taylor and Maclaurin Series

- Taylor and Maclaurin Polynomials
  - Recognize a Taylor series (5)
  - Find the Taylor polynomials for a function at a value (40)
- Estimating Function Values with Taylor and Maclaurin Series
  - Find the Maclaurin polynomials for a function (40)
  - Determine the error of an estimated function value using Taylor's theorem (5)
  - Estimate a function value using Taylor polynomials (40)
  - Estimate a trigonometric function value using Maclaurin polynomials (5)
- Representing Functions with Taylor and Maclaurin Series
  - Represent a function at a value with a Taylor series and determine the interval of convergence
  - Find the Maclaurin series for a function and show that the series converges

### 10.4 Working with Taylor Series

- Binomial Series
    - Find the binomial series for function (5)
    - Estimate a function value using a binomial series (5)
-

- Finding the Maclaurin Series that Represents a Function
  - Find the Maclaurin series for a trigonometric function (5)
  - Find the Maclaurin series for a logarithmic or exponential function (5)
  - Find a Maclaurin series by differentiating another series (5)
- Applications of the Taylor and Maclaurin Series
  - Evaluate a nonelementary definite integral using a Taylor series (5)
  - Approximate a probability using a Maclaurin series (5)
  - Approximate the period of a pendulum using a binomial series

## **Chapter 11: Parametric Equations and Polar Coordinates**

### 11.1 Parametric Equations

- Write Parametric Equations
  - Parameterize a curve (5)
  - Find the parametric equations for a line segment given an orientation (5)
- Eliminate the Parameter
  - Eliminate the parameter in linear equations (5)
  - Eliminate the parameter in polynomial and radical equations (8)
  - Eliminate the parameter in exponential and logarithmic equations (8)
  - Eliminate the parameter in trigonometric parametric equations (5)
- Graph Parametric Equations
  - Graph parametric equations by plotting points (5)
  - Graph trigonometric parametric equations by plotting points (5)
  - Use parametric equations in applications
- Cycloids
  - Graph a cycloid defined by parametric equations (5)
  - Determine the number of cusps on a hypocycloid (5)

### 11.2 Calculus of Parametric Curves

- Derivatives of Parametric Equations
  - Find the derivative of a curve defined by polynomial parametric equations
  - Find the derivative of a curve defined by trigonometric parametric equations
  - Find the equation of a line tangent to a parametrically defined curve
  - Find the second derivative of curve defined by parametric equations (5)
- Integrating Parametric Curves
  - Find the area under a curve defined by parametric equations (5)
  - Find the arc length of a curve defined by trigonometric parametric equations (5)
  - Find the surface area of a volume of revolution generated by revolving a parametrically defined curve

### 11.3 Polar Coordinates

- Defining Polar Coordinates
    - Locate points in a plane by using polar coordinates
    - Convert coordinates from polar form to rectangular form
    - Convert coordinates from rectangular form to polar form
  - Converting Equations Between Rectangular and Polar Forms
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- Convert a cartesian equation to polar form
- Convert a polar equation to cartesian form
- Graphs Using Polar Coordinates
  - Identify symmetry in polar equations and curves
  - Graph polar equations by plotting points and find zeros and maximum values for a polar equation

#### 11.4 Calculus in Polar Coordinates

- Area and Arc Length in Polar Coordinates
    - Find the area of a region between two polar curves
    - Find the arc length of a polar curve
    - Find the area of a region bounded by a polar curve
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