

## Survey of Mathematics | Table of Contents

### Chapter 1: Critical Thinking Skills

#### 1.1 Inductive and Deductive Reasoning

- Inductive and Deductive Reasoning
  - Understand and use inductive reasoning
  - Understand and use deductive reasoning

#### 1.2 Estimation

- Estimation by Rounding
  - Estimate a value by rounding a whole number
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- Estimation from Graphs/Figures
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#### 1.3 Problem Solving

- Problem Solving
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  - Solve an application problem by applying Polya's four step procedure

### Chapter 2: Sets

#### 2.1 Set Concepts

- Introduction to Sets and Set Builder Notation
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  - Represent a set using set builder notation
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- Types of Sets
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  - Distinguish between finite and infinite sets
- Subsets and Proper Subsets
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  - Determine the number of subsets and proper subsets in a given set

#### 2.2 Venn Diagrams and Set Operations

- Representing Sets with Venn Diagrams
    - Illustrate the universal set, a set, and complement of a set using a Venn diagram
    - Illustrate two sets using Venn diagram and set notation
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- Set Relationships
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  - Determine the intersection of two sets using Venn diagrams and set notation
  - Determine the union of two sets using Venn diagrams and set notation
- Set Operations
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### 2.3 Venn Diagrams with Three Sets and Verification of Equality of Sets

- Construct a Venn Diagram of Three Sets
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  - Identify and negate quantified statements
- Symbolic Representation of Statements
  - Identify logical connectives and compound statements
  - Represent and/or/not statements in symbolic form and English
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  - Represent conditional statements in symbolic form and English
  - Write biconditional statements in symbolic form and English
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### 3.2 Truth Tables for Negation, Conjunction, and Disjunction

- Introduction to Truth Tables
    - Construct a truth table for a statement with a conjunction and/or a negation and determine its truth value
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- Construct a truth table for a compound statement with a conjunction and disjunction and determine its truth value

### 3.3 Truth Tables for the Conditional and Biconditional

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- Drawing and Verifying Conclusions
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- Euler Diagrams and Syllogistic Arguments
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- Switching Circuits and Symbolic Logic
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### 4.1 Additive, Multiplicative, and Ciphred Systems of Numeration

- Additive Number Systems
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    - Convert between Roman Numerals and Hindu-Arabic Numerals
  - Multiplicative and Ciphred Number Systems
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#### 4.2 Place-Value or Positional-Value Numeration Systems

- Hindu-Arabic, Babylonian, and Mayan Numerals
  - Write a number in expanded form
  - Convert between Babylonian Numerals and Hindu-Arabic Numerals
  - Convert between Mayan Numerals and Hindu-Arabic Numerals

#### 4.3 Other Bases

- Bases Less Than 10
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  - Convert from base 10 to a system with a base less than 10
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  - Convert from a system with a base greater than 10 to base 10
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- Addition and Subtraction in Other Bases
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  - Perform subtraction in bases other than 10
- Multiplication and Division in Other Bases
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  - Perform division in bases other than 10

#### 4.5 Early Computational Methods

- Early Computational Methods
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  - Multiply two numbers using lattice multiplication
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### **Chapter 5: Number Theory and the Real Number System**

#### 5.1 Number Theory

- Prime Factorization, Greatest Common Factors, and Least Common Multiples
  - Understand and identify prime and composite numbers
  - Find the GCF and LCM of two or more numbers
  - Find the prime factorization of a number

#### 5.2 The Integers

- Introduction to Integers
    - Understand integers and find opposites of numbers
    - Order and compare integers
    - Understand and evaluate absolute value
-

- Add and Subtract Integers
  - Understand additive inverse
  - Understand distance in terms of absolute value
  - Add and subtract integers
- Multiply and Divide Integers
  - Multiply integers
  - Divide integers

### 5.3 The Rational Numbers

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  - Understand fractions and their models
  - Convert between fractions and mixed numbers
  - Find equivalent fractions
- Introduction to Decimals
  - Convert decimals to fractions
  - Convert fractions to decimals
  - Convert a repeating decimal to a fraction
- Multiply and Divide Fractions
  - Multiply fractions
  - Find reciprocals of fractions
  - Divide fractions
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  - Add and subtract fractions with unlike denominators
  - Add and subtract fractions in applications

### 5.4 Irrational Numbers, Radicals, and the Real Number System

- Irrational Numbers and Simplifying Radicals
  - Distinguish between rational and irrational numbers
  - Evaluate square roots
  - Use the product rule to simplify square roots
  - Use the quotient rule to simplify square roots
- Operations with Radicals
  - Add and subtract square roots
  - Rationalize denominators with a monomial denominator
  - Rationalize denominators using the conjugate

### 5.5 Real Numbers and Their Properties

- Properties of Real Numbers
    - Distinguish between natural numbers, whole numbers, and integers
    - Use the following properties of real numbers: commutative, associative, and distributive
-

## 5.6 Rules of Exponents and Scientific Notation

- Properties of Exponents
  - Understand exponent notation
  - Use the product rule of exponents
  - Use the quotient rule of exponents
  - Use the power rule of exponents
- Advanced Properties of Exponents
  - Use the negative and zero exponent rule
  - Find the power of a product
  - Find the power of a quotient
  - Simplify exponential expressions
- Scientific Notation
  - Convert between standard and scientific notation
  - Multiply and divide numbers in scientific notation
  - Multiply and divide numbers in scientific notation using a calculator

## 5.7 Arithmetic and Geometric Sequences

- Arithmetic Sequences
  - Write the terms of a sequence defined by an explicit formula
  - Find the common difference of an arithmetic sequence
  - Write terms of an arithmetic sequence
  - Write an explicit formula for an arithmetic sequence
- Arithmetic Series
  - Evaluate expressions using summation notation
  - Find the sum of a finite arithmetic series
- Geometric Sequences and Series
  - Find the common ratio of a geometric sequence
  - Write terms of a geometric sequence
  - Write an explicit formula for a geometric sequence
  - Find the sum of a finite geometric series

## **Chapter 6: Algebra, Graphs, and Functions**

### 6.1 Algebraic Expressions and the Order of Operations

- Simplifying Algebraic Expressions using Order of Operations
    - Perform calculations using order of operations
    - Identify constants and variables
    - Simplify algebraic expressions
  - Evaluating Algebraic Expressions
    - Evaluate algebraic expressions with a single variable
    - Evaluate algebraic expressions with two variables
-

## 6.2 Linear Equations

- Solve Linear Equations in One Variable
  - Identify identity, conditional, and inconsistent equations
  - Solve equations in one variable algebraically, variable just on one side
  - Solve equations in one variable algebraically, variable on both sides
  - Solve equations using cross multiplication

## 6.3 Formulas

- Using Formulas
  - Use a formula
  - Solve a formula for a given variable
- Exponential Formulas
  - Model exponential growth
  - Model exponential decay

## 6.4 Applications of Linear Equations in One Variable

- Solving Linear Equation Application Problems
  - Set up a linear equation to solve a real-world application
  - Translate verbal expressions into mathematical expressions
  - Use a formula to solve a real-world application

## 6.5 Variation

- Solving Variation Problems
  - Solve direct variation problems
  - Solve inverse variation problems
  - Solve problems involving joint variation
  - Solve combined variation problems

## 6.6 Linear Inequalities

- Linear Inequalities in One Variable
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  - Use properties of inequalities
  - Solve simple inequalities in one variable algebraically

## 6.7 Graphing Linear Equations

- Introduction to Graphing
    - Plot ordered pairs in a Cartesian coordinate system
    - Graph equations by plotting points
    - Graph a linear equation using the slope and the origin
  - Finding Linear Equations
    - Find equation of a line, in slope-intercept form, given slope and one point (point-slope formula)
    - Find equation, in slope-intercept form, of a line passing through two given points
    - Given slope and intercept, find the equation of a line and write it in standard form
-

- Vertical, Horizontal, Parallel, and Perpendicular Lines
  - Find the equation of vertical and horizontal lines
  - Given the equations of two lines, determine whether their graphs are parallel or perpendicular
  - Write the equation of a line parallel to a given line
  - Write the equation of a line perpendicular to a given line

#### 6.8 Linear Inequalities in Two Variables

- Linear Inequalities in Two Variables
  - Solve linear inequalities in two variables
  - Solve a linear inequality in two variables by graphing

#### 6.9 Solving Quadratic Equations

- Factoring Trinomials
  - Factor a trinomial
  - Factor a trinomial by grouping
  - Factor a perfect square trinomial
  - Factor a difference of squares
- Solving Quadratic Equations by Factoring
  - Solve quadratic equations by factoring, leading coefficient 1
  - Solve quadratic equations by factoring, leading coefficient  $> 1$
  - Use the discriminant to classify the solutions of a quadratic equation
  - Solve quadratic equations by using the quadratic formula

#### 6.10 Linear and Quadratic Functions and their Graphs

- Relations and Functions
    - Identify domain and range from a set of ordered pairs
    - Determine whether a relation represents a function
    - Use the vertical line test to identify functions
  - Graphs of Linear Functions
    - Represent a linear function in table form
    - Determine whether a linear function is increasing, decreasing, or constant
    - Graph linear functions
  - Graphs of Quadratic Function
    - Determine axis of symmetry and vertex of parabolas from a graph
    - Determine x- and y-intercepts of parabolas from a graph
  - Properties of Quadratic Functions
    - Find the direction a parabola opens and its axis of symmetry and vertex from the general form of its equation
    - Identify the axis of symmetry and vertex of a parabola from its equation in standard form
    - Find the domain and range of a quadratic function
    - Determine the maximum and minimum values of quadratic functions
-



## 6.11 Exponential Functions

- Identify and Evaluate Exponential Functions
  - Identify exponential functions
  - Evaluate exponential functions
  - Calculate continuous growth and decay
- Graphing Exponential Functions
  - Graph exponential functions
  - Graph exponential functions using transformations

## **Chapter 7: Systems of Linear Equations and Inequalities**

### 7.1 Systems of Linear Equations

- Solutions to Systems of Linear Equations
  - Determine whether an ordered pair is a solution to a system of equations
  - Solve systems of equations in two variables by graphing
  - Identify inconsistent and dependent systems of equations containing two variables, and express the solution of dependent equations

### 7.2 Solving Systems of Linear Equations by the Substitution and Addition Methods

- Solving Systems of Linear Equations
  - Solve systems of equations in two variables by substitution
  - Solve systems of equations in two variables by addition
- Applications of Systems of Linear Equations
  - Use systems of equations to investigate profits
  - Write and solve a system of equations in two variables from a word problem

### 7.3 Matrices

- Introduction to Matrices
  - Determine the order of a matrix and describe elements within a matrix
  - Add or subtract matrices
- Matrix Multiplication and the Identity Matrix
  - Multiply a matrix by a scalar
  - Find the sum or difference of scalar multiples
  - Multiply two matrices
  - Understand the identity matrix and how it relates to the inverse matrix

### 7.4 Solving Systems of Linear Equations by Using Matrices

- Solving Systems with Gaussian Elimination
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  - Use row operations to solve a system of linear equations in two variables

### 7.5 Systems of Linear Inequalities

- Solving Systems of Linear Inequalities
    - Solve a linear system of inequalities by graphing
-

## 7.6 Linear Programming

- Maximizing Profits using Linear Programming
  - Graph a feasible region given a set of constraints
  - Find the maximum value of an objective function given constraints by graphing
  - Solve application problems using linear programming

## **Chapter 8: The Metric System**

### 8.1 Basic Terms and Conversions Within the Metric System

- Measurements and the SI system
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  - Convert between units with different prefixes

### 8.2 Length, Area, and Volume

- Length, Area, and Volume
  - Identify units of length, area, or volume correctly for a given measurement
  - Calculate the volume of a shape given the formula

### 8.3 Mass and Temperature

- Mass and Temperature
  - Identify unit of mass correctly for a given situation
  - Convert between celsius and fahrenheit

### 8.4 Dimensional Analysis and Conversions to and from the Metric System

- Dimensional Analysis
  - Convert between non-metric units using dimensional analysis
  - Convert between non-metric units and metric units using dimensional analysis

## **Chapter 9: Geometry**

### 9.1 Points, Lines, Planes, and Angles

- Points, Lines, and Planes
  - Construct a line, line segment, and ray given two points
  - Find the intersection or union of two line segments, a ray and a line segment, or two rays
  - Identify planes
- Angles
  - Identify right, acute, obtuse, and straight angles
  - Understand supplementary and complementary angles
  - Understand alternate interior angles, alternate exterior angles, and corresponding angles

### 9.2 Polygons

- Polygons
    - Identify polygons given their properties
    - Use properties of similar polygons to solve for a missing side
    - Determine the measure of an angle using properties of polygons
-

- Triangles
  - Identify triangles given their properties
  - Use properties of triangles and right angles
  - Use properties of similar triangles to solve for a missing side or angle
  - Use the Pythagorean theorem

### 9.3 Perimeter and Area

- Area of Triangles and Quadrilaterals
  - Find the area of a rectangle
  - Find the area of a non-rectangular quadrilateral
  - Find the area of a triangle
  - Find the area of complex polygons
- Circles
  - Find the circumference and area of circles
  - Find the area of complex shapes including circles
- Perimeter and Applications
  - Calculate perimeter
  - Solve application problems involving area and perimeter

### 9.4 Volume and Surface Area

- Volume and Surface Area of Conventional Solids
  - Find the volume and surface area of rectangular solids
  - Find the volume of spheres, cylinders, and cones
  - Find the surface area of spheres and cylinders
- Volume and Surface Area of Other Solids
  - Find the volume and surface area of non-rectangular prisms
  - Find the volume and surface area of a pyramid
  - Use Euler's polyhedron formula to understand the relationship between vertices, edges, and faces in a polyhedron

### 9.5 Transformational Geometry, Symmetry, and Tessellations

- Reflections and Translations
    - Reflect a polygon across an axis
    - Translate a polygon given a translation vector
    - Perform a glide reflection on a polygon
  - Rotations
    - Rotate a polygon given an angle of rotation and a center of rotation outside the polygon
    - Rotate a polygon given an angle of rotation and a center of rotation inside the polygon
  - Symmetry
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    - Determine if a polygon has rotational symmetry about a point
-

**Chapter 10: Mathematical Systems**

## 10.1 Mathematical Systems

- Introduction to Mathematical Systems
  - Recognize mathematical systems
- Properties of a Mathematical System
  - Determine if a mathematical system is closed under an operation
  - Determine if a mathematical system is commutative and/or associative under an operation
  - Identify the identity element and the inverse of an element in a mathematical system
  - Determine the elements in the set, the binary operation, and the properties of a finite system defined by a given table
- Groups
  - Determine if a mathematical system is a group

## 10.2 Clock and Modular Arithmetic

- Clocks as a Mathematical System
  - Solve problems involving the finite mathematical system 'clock 12 arithmetic'
- Modular Arithmetic
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- Discounts and Income Tax
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- Simple Interest
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  - Calculate interest discounts on a discounted loan

## 11.3 Compound Interest

- Compound Interest
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#### 11.4 Annuities, Stocks, and Bonds

- Annuities
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  - Calculate the payment needed to achieve a determined future value
- Stocks
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  - Read a stock table

#### 11.5 Installment Loans, Amortization, and Credit Cards

- Mortgages and Loans
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  - Construct a loan amortization schedule
  - Choose the best installment loan plan
- Credit Cards
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  - Calculate the average daily balance of a credit card
  - Determine interest to be paid on a card's next billing date

### **Chapter 12: Probability and Counting Methods**

#### 12.1 The Fundamental Counting Principle

- The Fundamental Counting Principle
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  - Solve counting problems using the multiplication principle

#### 12.2 Permutations and Combinations

- Permutations
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  - Find the number of permutations of  $n$  distinct objects using the multiplication principle
  - Find the number of permutations of  $n$  distinct objects using a formula
  - Find the number of permutations of  $n$  non-distinct objects
- Combinations
  - Find the number of combinations using the formula
  - Find the number of subsets of a set

#### 12.3 Fundamentals of Probability

- Fundamentals of Probability
    - Compute the probability of equally likely outcomes
    - Compute the probability of equally likely outcomes in application
  - Probability with Permutations and Combinations
    - Compute probability involving permutations
    - Compute probability involving combinations
-

- The Complement Rule and Probability
  - Use the complement rule to compute probabilities
  - Compute the probability of an event happening at least once

#### 12.4 Events, Sample Spaces, and Odds

- Sample Spaces and Events
  - Determine the sample space of an experiment
  - Determine an event of an experiment
- Odds and Expected Value
  - Compute the expected value of an event
  - Compute odds using probability

#### 12.5 Conditional Probability, Independent, and Dependent Variables

- Independent Events
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  - Compute the probability of two independent events occurring
- Dependent Events and Conditional Probability
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  - Compute the probability of two or more dependent events occurring

#### 12.6 Random Variables, Probability Distributions and Expected Value

- Random Variables, Probability Distributions, and Expected Value
  - Calculate probability distribution
  - Calculate the expected value of a random variable

### **Chapter 13: Statistics**

#### 13.1 Sampling, Frequency Distributions, and Graphs

- Sampling and Parameters
  - Understand the definitions of population, sampling, statistic, parameter, and data
  - Identify stratified, cluster, systematic, and convenience sampling
  - Identify sampling errors and bias
  - Identify situations in which statistics can be misleading
- Frequency Distributions and Histograms
  - Construct and understand frequency tables for a set of data
  - Create and interpret histograms
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#### 13.2 Measures of Central Tendency

- Means and Medians
    - Find the mean of a set of data
    - Find the mean from a frequency table
    - Find the median of a set of data
-

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  - Determine whether the mean, median, or mode is the best measure of center for a data set

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### 13.4 The Normal Distribution, Margins of Error, and Skewness

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  - Compute z-scores and use them to compare values from different data sets
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- Percentiles, Quartiles, and Margins of Error
  - Find and interpret percentiles and quartiles of a data set
  - Calculate and interpret margin of error
- Problem Solving with the Normal Distribution
  - Standardize a normally distributed random variable
  - Calculate the mean and standard deviation of a standard normal distribution

### 13.5 Scatter Plots, Correlations, and Regression Lines

- Scatter Plots, Correlations, and Regression Lines
  - Understand the relationship between scatter plots and table and determine patterns
  - Find the linear regression equation given a list of data points
  - Find and interpret the correlation coefficient
  - Make predictions using a line of best fit
  - Find outliers in a data set

## **Chapter 14: Graph Theory**

### 14.1 Graphs, Paths, and Circuits

- Modeling Relationships with Graphs
    - Recognize equivalent graphs
    - Construct a simple graph to model relationships using information given
  - Basics of Graph Theory
    - Determine the degree of a vertex
    - Identify adjacent vertices
    - Recognize circuits
    - Distinguish between connected and disconnected graphs
-

## 14.2 Euler Paths and Euler Circuits

- Euler Paths and Euler Circuits
  - Identify Euler paths and circuits
  - Determine if a graph has a circuit using Euler's theorem
  - Solve application problems using Euler's theorem
  - Implement Fleury's algorithm to find Euler circuits when they exist

## 14.3 Hamilton Paths and Hamilton Circuits

- Hamilton Paths and Hamilton Circuits
  - Identify a Hamilton path and a Hamilton circuit for a given graph
  - Determine the number of Hamilton circuits in a graph
- Weighted Graphs
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## 14.4 Trees

- Trees
  - Identify a tree
  - Develop a spanning tree for a given graph
  - Use Kruskal's Algorithm to find a minimum spanning tree

## **Chapter 15: Voting and Apportionment**

### 15.1 Voting Methods

- Preference Tables and the Plurality Method
  - Identify elements of a preference table
  - Choose the winner of an election using the plurality method
- Choosing a Winner
  - Determine the winner of an election using the Borda count method
  - Decide the winner of an election using the plurality with elimination method
  - Select the winner of an election using the pairwise comparison method

### 15.2 Flaws of the Voting Methods

- Understanding the Fairness of a Voting System
  - Determine a voting systems fairness using the majority criterion
  - Determine a voting systems fairness using the head to head count criterion
  - Determine a voting system's fairness using the monotonicity criterion
  - Determine a voting system's fairness using the irrelevant alternatives criterion
  - Choose the best voting system for a given election

### 15.3 Apportionment Methods

- Standard Divisors and Standard Quotas
    - Determine the standard quota for a state given its population
    - Calculate the lower and upper quotas for a state given its population
-





- Apportionment Methods
  - Use Hamilton's method of determining apportionment
  - Use Jefferson's method of determining apportionment
  - Use Adam's method of determining apportionment
  - Use Webster's method of determining apportionment

#### 15.4 Flaws of the Apportionment Methods

- Flaws of the Apportionment Methods
    - Identify where the Alabama paradox occurs
    - Identify where the population paradox occurs
    - Identify where the new states paradox occurs
    - Choose the best apportionment system for a given region
-