Knewton Alta

Calculus Early Transcendentals | Table of Contents

Chapter 1: Functions and Graphs

1.1 Review of Functions

- Functions
  - Determine the domain and range of a relation and if a given relation is a function
    - Determine the domain and range of a relation given discrete values
    - Determine the domain and range of a relation given a graph
    - Determine if a relation is a function given discrete values
    - Determine if a relation is a function given a graph
  - Use functional notation to evaluate a function
    - Evaluate a function with a numerical argument
    - Evaluate a function with a variable argument
  - Determine the domain and range of a function
    - Determine the domain and range of a polynomial function
    - Determine the domain and range of a radical function
    - Determine the domain and range of a rational function
- Representing Functions
  - Find the zeros and intercepts of a function
    - Determine the zeros and intercepts of a linear function
    - Determine the zeros and intercepts of a rational function
    - Determine the zeros and intercepts of a radical function
    - Determine the zeros and intercepts of a function given a graph
  - Use a table to sketch a function
    - Complete a table of function values given a function
    - Sketch a function given a table of values
  - Determine the intervals where a function is increasing or decreasing
    - Determine the intervals where a function is increasing or decreasing given a function
    - Determine the intervals where a function is increasing or decreasing from a graph
- Combining Functions and Symmetry of Functions
  - Combine functions using mathematical operations
    - Combine functions using addition or subtraction
    - Combine functions using multiplication
    - Combine functions using division
    - Determine the domain of a combination of functions
  - Perform a composition of functions
    - Perform a composition of functions defined by formulas

        - Perform a composition of functions defined by formulas at a point
        - Determine the domain of a composition of functions defined by formulas
        - Perform a composition of functions defined by a table
    - Describe the symmetry properties of a function
        - Determine if a function is symmetric about an axis or the origin from a graph
        - Determine if a function is even or odd from a graph
        - Determine if a function is even or odd from a function

1.2 Basic Classes of Functions

- Linear Functions and Slope
    - Calculate the slope of a linear function
        - Calculate the slope given two points
        - Calculate the slope given a graph
        - Calculate the slope given a linear function
    - Find the equation of a line
        - Write the equation of a line in point-slope form given the slope and a point
        - Write the equation of a line in point-slope form given two points
        - Write the equation of a line in slope-intercept form given the slope and a point
        - Write the equation of a line in slope-intercept form given two points
        - Write the equation of a line given the x- and y-intercepts
    - Graph a linear equation
        - Identify the slope and intercept given a line in slope-intercept form
        - Graph a linear equation given in slope-intercept form
    - Use a linear function to model an application
        - Write a linear function to model an application
        - Interpret a linear function given an application
        - Graph a linear function to model an application
- Polynomials
    - Recognize key components of a polynomial
        - Recognize the degree of a polynomial
        - Find the roots of a factorable polynomial
        - Find the roots of a quadratic function using the quadratic formula
        - Describe the end-behavior of a polynomial function
    - Graph a polynomial function
        - Graph a polynomial function given information about the function
        - Graph a polynomial function
    - Use polynomials to model an application
        - Interpret a polynomial function given an application
        - Graph a polynomial function to model an application
    - Determine whether a function is algebraic or transcendental

- Piecewise Functions and Transformations of Functions
    - Use piecewise functions
        - Evaluate a piecewise function
        - Graph a piecewise defined function
        - Write a piecewise function to model an application
    - Perform a transformation of functions
        - Transform a function by performing a horizontal or vertical shift
        - Transform a function by performing a vertical or horizontal compression
        - Transform a function by performing a reflection about the x-axis or y-axis

1.3 Trigonometric Functions

- Radian Measure and the Trigonometric Functions
    - Understand the unit circle and convert angle measures between degrees and radians
        - Convert an angle measure from degrees to radians
        - Convert an angle measure from radians to degrees
        - Find a coterminal angle
        - Find the reference angle for a given angle
    - Use the circular definitions of the basic trigonometric functions
        - Find the exact value of a sine or cosine function given an angle measure
        - Find the exact value of a tangent or cotangent function given an angle measure
        - Find the exact value of a secant or cosecant function given an angle measure
        - Relate points on the unit circle to a trigonometric ratio
    - Use the triangular definitions of the basic trigonometric functions
        - Find a missing side length of a right triangle given two sides
        - Find a trigonometric ratio given side lengths of a right triangle
        - Find a missing side length of a right triangle given a side and an angle
- Trigonometric Equations and Identities
    - Solve trigonometric equations
        - Solve a trigonometric equation involving sine or cosine
        - Solve a trigonometric equation involving tangent or cotangent
        - Solve a trigonometric equation involving secant or cosecant
        - Solve an application problem involving trigonometric equations
    - Use trigonometric identities to simplify and rewrite expressions
        - Use the reciprocal trigonometric identities to write an expression in terms of sine and cosine
        - Use the pythagorean trigonometric identities to simplify an expression
        - Use the addition and subtraction formulas to rewrite an expression
        - Use the double-angle formulas to simplify an expression
- Graphing Trigonometric Functions
    - Graph trigonometric functions

- Graph a trigonometric equation involving sine or cosine
- Graph a trigonometric equation involving tangent or cotangent
- Graph a trigonometric equation involving secant or cosecant

1.4 Inverse Functions

- Finding Inverse Functions
  - Determine if a function is one-to-one
    - Determine if a function is one-to-one given a set of ordered pairs
    - Determine if a function is one-to-one given a table of values
    - Determine if a function is one-to-one given a graph using the horizontal line test
  - Find the inverse of a function
    - Determine if two functions are inverses of each other given a graph
    - Graph the inverse of a function
    - Determine if two functions are inverses of each other by performing a composition of functions
    - Find the inverse of a function algebraically
  - Understand domain and range of an inverse function
    - Find the domain and range of the inverse of a polynomial function
    - Find the domain and range of the inverse of a radical function
    - Find the domain and range of the inverse of a rational function
    - Restrict the domain of a function to find an inverse function
- Inverse Trigonometric Functions
  - Evaluate inverse trigonometric functions
    - Determine the domain of inverse trigonometric functions
    - Identify the graphs of inverse trigonometric functions
    - Evaluate inverse trigonometric functions
    - Use inverse trigonometric functions to solve trigonometric equations

1.5 Exponential and Logarithmic Functions

- Exponential Functions
  - Evaluate an exponential function
    - Evaluate an exponential function at an integer value
    - Evaluate an exponential function at a rational value
    - Evaluate an exponential function in an application
  - Use the properties of exponents to simplify expressions
    - Use the properties of exponents to simplify expressions with a single variable
    - Use the properties of exponents to simplify expressions with a single variable and rational exponents
    - Use the properties of exponents to simplify algebraic expressions with two variables
  - Relate an exponential function to its graph

        - Determine the domain, range, and asymptotes of an exponential function
        - Graph an exponential function
        - Graph an exponential function using transformations
        - Find the equation for an exponential function given its graph
    - Use exponential functions and base e in applications
        - Write an exponential function to model an application
        - Interpret an exponential function given an application
        - Graph an exponential function to model an application
- Logarithmic Functions
    - Relate a logarithmic expression to an exponential expression and use properties of logarithms
        - Write an exponential expression in logarithmic form
        - Write a logarithmic expression in exponential form
        - Use the properties of logarithms to condense a logarithmic expression
        - Use the properties of logarithms to expand a logarithmic expressions
        - Evaluate a logarithmic expression
    - Relate a logarithmic function to its graph
        - Determine the domain, range, and asymptotes of a logarithmic function
        - Graph a logarithmic function
        - Graph a logarithmic function using transformations
        - Find the equation for a logarithmic function given its graph
- Exponential and Logarithmic Equations
    - Solve exponential and logarithmic equations
        - Solve a factorable exponential equation
        - Solve an exponential equation using logarithms
        - Solve a logarithmic equation by exponentiating
        - Solve a logarithmic equation by using the properties of logarithms
- Hyperbolic Functions
    - Evaluate a hyperbolic function
        - Evaluate a hyperbolic function involving hyperbolic sine or cosine
        - Evaluate a hyperbolic function involving hyperbolic tangent or cotangent
        - Evaluate a hyperbolic function involving hyperbolic secant or cosecant
        - Given the value of one hyperbolic function find the value of another
    - Graph a hyperbolic function
        - Graph a hyperbolic function involving hyperbolic sine or cosine
        - Graph a hyperbolic function involving hyperbolic tangent or cotangent
        - Graph a hyperbolic function involving hyperbolic secant or cosecant
- Inverse Hyperbolic Functions
    - Evaluate an inverse hyperbolic function

        - Evaluate an inverse hyperbolic function involving inverse hyperbolic sine or cosine
        - Evaluate an inverse hyperbolic function involving inverse hyperbolic tangent or cotangent
        - Evaluate an inverse hyperbolic function involving inverse hyperbolic secant or cosecant
    - Graph an inverse hyperbolic function
        - Graph an inverse hyperbolic function involving inverse hyperbolic sine or cosine
        - Graph an inverse hyperbolic function involving inverse hyperbolic tangent or cotangent
        - Graph an inverse hyperbolic function involving inverse hyperbolic secant or cosecant

Chapter 2: Limits

2.1 A Preview of Calculus

- A Preview of Differential and Integral Calculus
    - Preview the area problem by using rectangles to approximate area
        - Find the slope of secant lines
        - Estimate the slope of the tangent line to a point using secant lines
        - Find the average velocity
        - Estimate the instantaneous velocity at a given time using average velocities
    - Preview the area problem by using rectangles to approximate area

2.2 The Limit of a Function

- The Intuitive Definition of a Limit
    - Understand the limit of a function and evaluate a limit from a table
        - Use correct notation to describe the limit of a function
        - Use a table of values to estimate the limit of a function or to identify when the limit does not exist
    - Evaluate limits graphically
- One-Sided Limits
    - Evaluate a one-sided limit from a table or a graph
        - Evaluate a one-sided limit from a table
        - Evaluate a one-sided limit from a graph
    - Explain the relationship between one-sided and two-sided limits
        - Determine when a two-sided limit exists by analyzing one-sided limits from a table
        - Determine when a two-sided limit exists by analyzing one-sided limits from a graph
- Infinite Limits

- Find an infinite limit
    - Use correct notation to describe the infinite limit of a function
    - Determine an infinite limit from a table
    - Determine an infinite limit from a graph
- Define a vertical asymptote

2.3 The Limit Laws

- Evaluating Limits with the Limit Laws
    - Use the limit laws to evaluate the limit of a function
        - Use the sum, difference, or constant multiple law for limits to evaluate a limit
        - Use the quotient or power law for limits to evaluate a limit
        - Use the product or root law for limits to evaluate a limit
- Evaluate a Basic Limit
    - Find the limit of a polynomial or rational function using limit laws and direct substitution
        - Find the limit of a polynomial function
        - Find the limit of a rational function
    - Use one-sided limits to evaluate a limit using limit laws
        - Evaluate a one-sided limit using the limit laws
        - Find the limit of a piecewise function
        - Find the limit of an absolute value function
        - Find the limit of a step function
- Additional Limit Evaluation Techniques
    - Evaluate the limit of a rational function by simplification
        - Find the limit of a function with removable discontinuities by factoring
        - Find the limit of a function with removable discontinuities through expansion
        - Find the limit of a function by adding two rational expressions
    - Evaluate the limit of a function with complex fractions or rational functions that contain radicals
        - Find the limit of a rational function that contains radicals
        - Find the limit of a function with complex fractions
- The Squeeze Theorem and Limits of Trigonometric Functions
    - Evaluate the limit of a function by using the squeeze theorem
        - Demonstrate a conceptual understanding of the squeeze theorem
        - Use the squeeze theorem to find the limit of a trigonometric function
    - Evaluate the limit of a trigonometric function by direct substitution
    - Evaluate the limit of a trigonometric function using special limits
        - Evaluate trigonometric limits using special limits
        - Evaluate trigonometric limits using special limits and trigonometric identities

2.4 Continuity

- Continuity at a Point

- Explain the three conditions for continuity at a point
    - Determine which condition of the definition of continuity fails given a graph
    - Determine which condition of the definition of continuity fails given a function
- Distinguish between three kinds of discontinuity
    - Distinguish between types of discontinuity given a graph
    - Distinguish between types of discontinuity given a function
- Determine the continuity of a piecewise function
    - Determine whether a piecewise function is continuous at a given value
    - Determine the value that makes a piecewise function continuous
    - Determine the intervals on which a piecewise function is continuous from a graph

- Continuity over an Interval and the Intermediate Value Theorem
    - Determine where an algebraic function is continuous
        - Determine where a polynomial or rational function is continuous
        - Determine where a radical function is continuous
    - Determine the limit of a composite function using the composite function theorem
        - Determine the limit of a composite trigonometric function
        - Determine the limit of a composite exponential or logarithmic function
        - Determine the limit of a composite inverse trigonometric function
    - Understand and apply the Intermediate Value Theorem
        - Determine if the intermediate value theorem applies
        - Determine how many roots a continuous function may have given a table
        - Use the intermediate value theorem given an equation

2.5 The Precise Definition of a Limit

- Use the Epsilon-Delta Definition of the Limit to Prove Statements about Limits
    - Understand the notation in finding the formal definition of a limit
        - Given a limit statement, write the corresponding epsilon-delta statement
        - Given the epsilon-delta statement of a limit, write the corresponding limit statement
        - Use graphs to demonstrate an understanding of the epsilon-delta definition of limits
    - Determine a delta for a given or arbitrary epsilon given a function
        - Determine a delta for a given epsilon for a linear function
        - Determine a delta for an arbitrary epsilon given a linear function
        - Determine a delta for an arbitrary epsilon given a quadratic function
    - Use the epsilon-delta definition of limits to understand one-sided and infinite limits
        - Prove a statement about a limit from one side
        - Prove a statement about infinite limits

Knewton Alta

Chapter 3: Derivatives

3.1 Defining the Derivative

- Tangent Lines
  - Find the slope of a secant line
    - Find the slope of the secant line given a function
    - Find the slope of the secant line given a table or graph
    - Find the slope of the secant line given a function and variable intervals
  - Find the slope of a tangent line using the first limit definition
    - Find the slope of a tangent line given a linear function using the first limit definition
    - Find the slope of a tangent line given a quadratic function using the first limit definition and factoring
    - Find the slope of a tangent line given a rational function using the first limit definition and complex fractions
    - Find the slope of a tangent line given a radical function using the first limit definition
  - Find the slope of a tangent line using the second limit definition
    - Find the slope of a tangent line given a linear function using the second limit definition
    - Find the slope of a tangent line given a quadratic function using the second limit definition and expansion
    - Find the slope of a tangent line given a rational function using the second limit definition and complex fractions
    - Find the slope of a tangent line given a radical function using the second limit definition
  - Find the equation of a tangent line using a limit definition
    - Find the equation of a tangent line given a linear function using the second limit definition
    - Find the equation of a tangent line given a quadratic function using the second limit definition and expansion
    - Find the equation of a tangent line given a rational function using the second limit definition and complex fractions
    - Find the equation of a tangent line given a radical function using the second limit definition
- The Derivative of a Function at a Point
  - Identify the derivative as the limit of a difference quotient
    - Identify the function f and number a given the limit definition of a derivative for a polynomial function

        - Identify the function f and number a given the limit definition of a derivative for a radical function
        - Identify the function f and number a given the limit definition of a derivative for a rational function
        - Write the limit definition of a derivative given a function and a value
    - Estimate the value of a derivative at a point
    - Calculate the derivative of a given function at a point using a limit
        - Find the value of the derivative at a point given a linear function using the definition of a derivative
        - Find the value of the derivative at a point given a quadratic function using the definition of a derivative
        - Find the value of the derivative at a point given a rational function using the definition of a derivative
        - Find the value of the derivative at a point given a radical function using the definition of a derivative
- Velocities and Rates of Change
    - Use the concept of a derivative to find an instantaneous velocity
        - Estimate the velocity at a time given a table of values
        - Use the limit definition of a derivative to find the instantaneous velocity given a position function
    - Use instantaneous rates of change in other contexts
        - Estimate the instantaneous rate of change in different contexts given a table of values
        - Determine the rate of change of temperature given a function
        - Determine the rate of change in a business context given a function

3.2 The Derivative as a Function

- Derivative Functions and their Graphs
    - Define the derivative function of a given function
        - Find the derivative function of a linear or quadratic function using limits
        - Find the derivative function of a radical function using limits
        - Find the derivative function of a rational function using limits
    - Graph the derivative of a function
        - Estimate the value of a derivative at a point on a graph using a tangent line
        - Determine the open intervals where the derivative is positive or negative from a graph
        - Identify the graph of a derivative function given a function
        - Graph a derivative function given a function
- Derivatives and Continuity
    - Understand differentiability and when a function does not have a derivative

        - State the connection between derivatives and continuity
        - Describe three conditions for when a function does not have a derivative
    - Determine the differentiability of piecewise functions
        - Determine if a piecewise function is differentiable
        - Determine if a piecewise function is differentiable given a graph
        - Find the constants that make a piecewise function differentiable
- Higher-Order Derivatives
    - Determine the second derivative of a function using the limit definition
        - Interpret the meaning of a higher-order derivative
        - Find the second derivative of a polynomial function
    - Graph higher-order derivative functions
        - Identify the graph of a higher-order derivative function given a function
        - Graph a higher-order derivative function given a function
    - Relate higher-order derivatives to acceleration
        - Interpret the meaning of a higher-order derivative with respect to a position function
        - Find the acceleration function given a position function using limits

3.3 Differentiation Rules

- The Basic Differentiation Rules
    - Use the constant, constant multiple, and power rules
        - Find the derivative of a constant function
        - Find the derivative of a function using the power rule for monomials
        - Find the derivative of a function using the constant multiple rule with an integer coefficient
        - Find the derivative of a function using the constant multiple rule with a fractional coefficient
    - Apply the sum and difference rules to combine derivatives
        - Find the derivative of a function using the sum of functions
        - Find the derivative of a function using the difference of two functions
        - Find the derivative of a function using both the sum and difference of functions
        - Find the derivative of a function using both the sum and difference of functions with fractional coefficients
        - Evaluate the derivative of a function at a point using both the sum and difference of functions with fractional coefficients
        - Find the derivative of a function using sum and difference rules and simplification
    - Find the equation of a tangent line using basic derivative rules
        - Find the equation of a line tangent to a monomial function
        - Find the equation of a line tangent to a polynomial function
        - Graph a line tangent to a polynomial function

        - Find the slope of the line tangent to a polynomial function
        - Find the equation of a line normal to a polynomial function
- The Product and Quotient Rules
    - Use the product rule to find the derivative of a product of functions
        - Apply the product rule given information of two functions at a point
        - Apply the product rule given one function and information of another function at a point
        - Find the derivative of the product of two functions
        - Find the value of the derivative at a point of the product of two functions
    - Use the quotient rule to find the derivative of a quotient of functions
        - Apply the quotient rule given information of two functions at a point
        - Apply the quotient rule given one function and information of another function at a point
        - Find the derivative of the quotient of two functions
        - Find the value of the derivative at a point of the quotient of two functions
    - Combine the product and quotient rules
        - Combine the product or quotient rule with the sum and difference rules
        - Find the derivative of the product of three functions
        - Combine the product and the quotient rule
- Extensions of the Power Rule
    - Extend the power rule to functions with negative exponents
        - Find the derivative of a monomial function with negative exponents
        - Find the derivative of a function with negative exponents
        - Rewrite a rational function to use the power rule to find its derivative
    - Extend the power rule to functions with rational exponents
        - Find the derivative of a monomial function with rational exponents
        - Find the derivative of a function with rational exponents
        - Find the derivative of a function with negative rational exponents
        - Rewrite a function with radicals to use the power rule to find its derivative
        - Rewrite a function with radicals to use the power rule and simplify to find its derivative
    - Use the extensions of the power rule to find tangent lines
        - Find the value of the derivative of a point using the extension of the power rule
        - Find the equation of the line tangent to a point using the extension of the power rule
        - Graph the equation of the line tangent to a point using the extension of the power rule

3.4 Derivatives as Rates of Change

- Motion along a Line

- Find the time that a moving object has a certain position given the position function
- Find the velocity function given the position function
    - Find the velocity function given the position function
    - Interpret the movement of an object given its position function
- Find the acceleration function given a position function
- Interpret the relationship between the velocity function and acceleration function

- Changes in Cost and Revenue
    - Use cost, revenue, and profit functions
        - Find the cost of producing a given number of items given a cost function
        - Create a revenue function given a price function
        - Create a profit function given a price function and cost function
    - Use derivatives to calculate marginal cost and revenue in a business situation
        - Find a marginal cost function
        - Find a marginal revenue function
        - Find a marginal profit function

3.5 Derivatives of Trigonometric Functions

- Derivatives of Trigonometric Functions
    - Find the derivative of a sine or cosine function
        - Find the derivative of a sine or cosine function using sum, difference, or constant multiple rules
        - Find the derivative of a sine or cosine function combined with the product rule
        - Find the derivative of a sine or cosine function combined with the quotient rule
    - Find the derivative of other trigonometric functions
        - Find the derivative of a tangent or cotangent function using sum, difference, or constant multiple rules
        - Find the derivative of a secant or cosecant function using sum, difference, or constant multiple rules
        - Find the derivative of a combination of trigonometric functions using sum, difference, or constant multiple rules
        - Find the derivative of a combination of trigonometric functions using the product rule
        - Find the derivative of a combination of trigonometric functions using the quotient rule
    - Find a higher-order derivative of a sine or cosine function
        - Find a higher-order derivative of a sine function
        - Find a higher-order derivative of a cosine function
    - Find the line tangent to a trigonometric function
        - Find the line tangent to a sine or cosine function
        - Find the line tangent to a tangent or cotangent function

- Find the line tangent to a secant or cosecant function

3.6 The Chain Rule

- The Chain Rule
  - Use the chain rule with the power rule
    - Decompose a composite function
    - Use the chain rule given decomposed functions
    - Use the chain rule given a graph
    - Use the chain rule with a polynomial function to some whole number power
    - Use the chain rule with a polynomial function to some negative integer power
    - Use the chain rule with a polynomial function to some rational power
    - Use the chain rule given functional values
  - Use the chain rule with the product or quotient rule
    - Use the chain rule with a polynomial function and the product rule
    - Use the chain rule with a polynomial function and the product rule with a radical
    - Use the chain rule with a polynomial function and the quotient rule
    - Use the chain rule with a polynomial function and the quotient rule with a radical
  - Use the chain rule with trigonometric functions
    - Use the chain rule with a sine or cosine function to some power
    - Use the chain rule with a sine or cosine function of some power
    - Differentiate a composite of three functions involving sine or cosine
    - Use the chain rule with a tangent or cotangent function
    - Use the chain rule with a secant or cosecant function

3.7 Derivatives of Inverse Functions

- Find the Derivative of an Inverse Function
  - Estimate the value of the derivative of an inverse function given the graph of a function
  - Calculate the derivative of an inverse function
    - Find an inverse derivative function value for a polynomial function
    - Find an inverse derivative function value for a rational function
    - Find an inverse derivative function for a radical function
  - Find the equation of the line tangent to an inverse function
    - Find an inverse derivative function value for a polynomial function
    - Graph the equation of the line tangent to the inverse of a polynomial function
    - Find an inverse derivative function value for a rational function
    - Find an inverse derivative function for a radical function
- Find the Derivative of an Inverse Trigonometric Function
  - Write the algebraic form of an inverse trigonometric function
  - Find the derivative of an inverse trigonometric function
    - Find the derivative of an inverse trigonometric function at a value
    - Find the derivative of an inverse sine or cosine function using the chain rule

- Find the derivative of other inverse trigonometric functions using the chain rule

3.8 Implicit Differentiation

- Implicit Differentiation
  - Find the derivative of a function using implicit differentiation
    - Find the derivative of a function using implicit differentiation and the power rule
    - Find the derivative of a function using implicit differentiation and the extension of the power rule
  - Find the derivative of a function using implicit differentiation and the product rule
    - Find the derivative of a function using implicit differentiation and the product rule
    - Find the derivative of a function using implicit differentiation and the product rule with a sine or cosine expression
  - Use implicit differentiation to find a second derivative
  - Use implicit differentiation to find the equation of a tangent line
    - Find the value of the derivative at a point using implicit differentiation and the power rule
    - Find the value of the derivative at a point using implicit differentiation and the product rule
    - Find the equation of the tangent line at a point using implicit differentiation and the power rule
    - Find the equation of the tangent line at a point using implicit differentiation and the product rule

3.9 Derivatives of Exponential and Logarithmic Functions

- Derivatives of Exponential and Logarithmic Functions with Base e
  - Find the derivative of an exponential function with base e
    - Find the derivative of an exponential function with the product rule
    - Find the derivative of an exponential function with the chain rule
    - Find the derivative of an exponential function with the product rule and the chain rule
    - Find the derivative of an exponential function with the quotient rule
    - Find the derivative of an exponential function with a trigonometric function
  - Find the derivative of a natural logarithmic function
    - Find the derivative of a natural logarithmic function with the product rule
    - Find the derivative of a natural logarithmic function with the chain rule
    - Find the derivative of a natural logarithmic function with the product rule and the chain rule
    - Find the derivative of a natural logarithmic function with the quotient rule
    - Find the derivative of a natural logarithmic function with a trigonometric function
  - Find the derivative of a natural logarithmic function using properties of logarithms

        - Find the derivative of a natural logarithmic function using properties of logarithms
        - Find the derivative of a natural logarithmic function with a trigonometric function using properties of logarithms
- Derivatives of Exponential and Logarithmic Functions of Any Base
    - Find the derivative of an exponential function of any base
        - Find the derivative of an exponential function of any base with the product rule
        - Find the derivative of an exponential function of any base with the chain rule
        - Find the derivative of an exponential function of any base with the product and chain rules
        - Find the derivative of an exponential function of any base with the quotient rule
        - Find the derivative of an exponential function of any base with a trigonometric function
    - Find the derivative of a logarithmic function of any base
        - Find the derivative of a logarithmic function of any base with the chain rule
        - Find the derivative of a logarithmic function of any base with the product rule
        - Find the derivative of a logarithmic function of any base with the quotient rule
        - Find the derivative of a logarithmic function of any base with a trigonometric function
    - Find the derivative of a logarithmic function of any base using properties of logarithms
        - Find the derivative of a logarithmic function of any base using properties of logarithms
        - Find the derivative of a logarithmic function of any base with a trigonometric function using properties of logarithms
- Logarithmic Differentiation
    - Use logarithmic differentiation to find a derivative
        - Use logarithmic differentiation and the power property of logarithms to find the derivative of a function
        - Use logarithmic differentiation and the power property of logarithms to find the derivative of a function with a trigonometric function
    - Use logarithmic differentiation and properties of logarithms to find a derivative
        - Use logarithmic differentiation and all of the properties of logarithms to find the derivative of a function
        - Use logarithmic differentiation and all of the properties of logarithms to find the derivative of a function with a trigonometric function

Chapter 4: Applications of Derivatives

4.1 Related Rates

- Introduction to Related Rates and Related Rates with Volume or Area Problems

- Express changing quantities in terms of derivatives
- Use related rates given an abstract equation
- Use related rates to solve problems involving volume
    - Use related rates to solve problems involving volume
    - Use related rates to solve problems involving volume and similar figures
- Use related rates to solve problems involving area
    - Use related rates to solve problems involving area
    - Use related rates to solve problems involving area given two rates of change
- Related Rates in Other Applications
    - Use related rates to solve problems involving distance
        - Use related rates to solve problems involving distance with one length staying constant
        - Use related rates to solve problems involving distance with all lengths changing
    - Use related rates to solve problems involving angles or shadows
        - Use related rates to solve problems involving changing angles
        - Use related rates to solve problems involving shadows

4.2 Linear Approximations and Differentials

- Linear Approximation of a Function at a Point
    - Describe the linear approximation to a function at a point
        - Graphically represent a linear approximation
        - Find the linear approximation for a polynomial function
        - Find the linear approximation for a radical function
        - Find the linear approximation for a rational function
        - Find the linear approximation for a sine or cosine function
    - Use a linear approximation to estimate a function value
        - Use a linear approximation to estimate a polynomial function value
        - Use a linear approximation to estimate a radical function value
        - Use a linear approximation to estimate a rational function value
        - Use a linear approximation to estimate a sine or cosine function value
- Differentials and Calculating Error
    - Find the differential of a function
        - Find the differential for a polynomial function
        - Find the differential for a radical function
        - Find the differential for a rational function
        - Find the differential for a sine or cosine function
    - Compute a differential
        - Compute a differential for a polynomial function
        - Compute a differential for a radical function
        - Compute a differential for a rational function

        - Compute a differential for a sine or cosine function
    - Represent a differential graphically
        - Represent a differential graphically
        - Represent a differential graphically and interpret its values
    - Use a differential to find a change in a geometric context
        - Use a differential to find a change in volume
        - Use a differential to find a change in surface area
    - Calculate the relative error and percentage error in using a differential approximation
        - Estimate the propagated error using differentials
        - Find the relative or percentage error using differentials

4.3 Maxima and Minima

- Critical Points and Extrema
    - Define absolute extrema and understand the extreme value theorem
        - Determine if the extreme value theorem applies for a given algebraic function
        - Determine if the extreme value theorem applies for a given trigonometric function
        - Determine if the extreme value theorem applies for a given exponential or logarithmic function
        - Determine from a graph if a function will have absolute extrema
    - Use derivatives to find critical points of a function
        - Identify local extrema from a graph
        - Find any critical points of a polynomial function
        - Find any critical points of a function with rational exponents
        - Find any critical points of a rational function
        - Find any critical points of a sine or cosine function
    - Find absolute extrema
        - Identify absolute extrema on a closed interval given a polynomial function
        - Identify absolute extrema on a closed interval given a function with rational exponents
        - Identify absolute extrema on a closed interval given a rational function
        - Identify absolute extrema on a closed interval given a sine or cosine function

4.4 The Mean Value Theorem

- Rolle’s Theorem and the Mean Value Theorem for Derivatives
    - Apply Rolle's theorem
        - Determine if Rolle's theorem can be used for a function on a given interval
        - Represent Rolle's theorem graphically
        - Find all values c on a given interval that satisfy the conclusion of Rolle's theorem
    - Find values of c guaranteed by the Mean Value Theorem
        - Represent the Mean Value Theorem graphically

        - Find values of c guaranteed by the Mean Value Theorem for a polynomial function
        - Find values of c guaranteed by the Mean Value Theorem for a function with rational exponents
        - Find values of c guaranteed by the Mean Value Theorem for a rational function
    - Use the Mean Value Theorem and its corollaries to solve problems
    - Use the Mean Value Theorem in applications

4.5 Derivatives and the Shape of a Graph

- The First Derivative Test
    - Understand the relationship between the graph of a function and the sign of its first derivative
        - Determine the sign of a derivative function given a graph of a function
        - Determine when a function is increasing or decreasing given the graph of its derivative
        - Determine when a function is increasing or decreasing given information about its derivative
    - Use the first derivative test to find local extrema of a function given its graph
    - Use the first derivative test to find local extrema of an algebraic function
        - Use the first derivative test and information about a function to find local extrema
        - Use the first derivative test to find local extrema of a polynomial function
        - Use the first derivative test to find local extrema of a function with rational exponents or radicals
        - Use the first derivative test to find local extrema of a function with a rational function
    - Use the first derivative test to find local extrema of a transcendental function
        - Use the first derivative test to find local extrema of a sine or cosine function
        - Use the first derivative test to find local extrema of an exponential function
        - Use the first derivative test to find local extrema of a logarithmic function
- Concavity and Points of Inflection
    - Understand the relationship between the graph of a function and the sign of its second derivative
        - Determine the concavity of a function given its graph
        - Determine the concavity of a function given the graph of its second derivative
        - Determine the concavity of a function given the graph of its first derivative
    - Find inflection points and intervals of concavity given a function
- The Second Derivative Test
    - Use the second derivative test to find local extrema of an algebraic function

        - Use the second derivative test and information about a function to find local extrema
        - Use the second derivative test to find local extrema of a polynomial function
        - Use the second derivative test to find local extrema of a function with rational exponents or radicals
    - Use the second derivative test to find local extrema of a transcendental function
        - Use the second derivative test to find local extrema of a sine or cosine function
        - Use the second derivative test to find local extrema of an exponential function
        - Use the second derivative test to find local extrema of a logarithmic function
    - Use both the first and second derivatives of a function to determine the basic shape of a graph

4.6 Limits at Infinity and Asymptotes

- Limits at Infinity
    - Understand the meaning of a limit at infinity and horizontal asymptotes
        - Determine the limit at infinity of a function given the graph of a function
        - Determine the limit at infinity of a function using a table of values
        - Find the equation of the horizontal asymptote for a given function
    - Determine the limit at infinity of a polynomial function
        - Determine the limit at infinity of a power function
        - Determine the limit at infinity of a polynomial function
    - Determine the limit at infinity of a rational or other algebraic function
        - Determine the limit at infinity of a rational function
        - Determine the limit at infinity of a rational function involving a radical
        - Determine the limit at infinity of a function involving a radical
    - Determine the limit at infinity of a transcendental function
        - Determine the limit at infinity of a sine or cosine function
        - Determine the limit at infinity of another trigonometric function
        - Determine the limit at infinity of an exponential function
        - Determine the limit at infinity of a logarithmic function
        - Determine the limit at infinity of an inverse trigonometric function
    - Use the precise definition of a limit at infinity
        - Use the precise definition of a limit at infinity to write a statement proving a limit
        - Represent the precise definition of a limit at infinity graphically
- Oblique Asymptotes and Curve Sketching
    - Find oblique asymptotes of a function
        - Determine when a function will have an oblique asymptote
        - Find the equation of the oblique asymptote for a given function
        - Graph the oblique asymptote of a function
    - Sketch the graph of a function

        - Draw a graph of a polynomial
        - Draw a graph of a rational function
        - Draw a graph of a rational function with an oblique asymptote
        - Draw a graph of a function with a cusp

4.7 Applied Optimization Problems

- Optimization Problems in a Geometric Context
    - Solve optimization problems involving area or volume
        - Solve an optimization problem involving area
        - Solve an optimization problem involving volume
    - Minimize surface area
    - Minimize distance or travel time
        - Minimize travel time using optimization
        - Minimize a distance using optimization
- Optimization Problems in Other Contexts
    - Solve optimization problems in the abstract
        - Minimize the distance from a function to a point
        - Maximize the area of an inscribed rectangle
        - Maximize or minimize quantities given an expression with two variables
    - Solve optimization problems in a business or economics context
        - Use a demand function to maximize revenue
        - Maximize profit given a revenue function and cost function

4.8 L'Hospital's Rule

- L'Hospital's Rule with an Indeterminate Quotient
    - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a quotient of the form (0/0)
        - Apply L'Hospital's rule in the 0/0 case
        - Apply L'Hospital's rule in the 0/0 case with a sine or cosine function
        - Apply L'Hospital's rule in the 0/0 case with another trigonometric function
        - Apply L'Hospital's rule in the 0/0 case with an exponential function
        - Apply L'Hospital's rule in the 0/0 case with a logarithmic function
    - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a quotient of the form (infinity/infinity)
        - Apply L'Hospital's rule in the (infinity/infinity) case
        - Apply L'Hospital's rule in the (infinity/infinity) case with an exponential function
        - Apply L'Hospital's rule in the (infinity/infinity) case with a logarithmic function
- L'Hospital's Rule with an Indeterminate Product, Difference, or Power
    - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a product
        - Apply L'Hospital's rule in the 0*infinity case with an exponential function
        - Apply L'Hospital's rule in the 0*infinity case with a logarithmic function

        - Apply L'Hospital's rule in the 0*infinity case with a sine or cosine function
        - Apply L'Hospital's rule in the 0*infinity case with another trigonometric function
    - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a difference
        - Apply L'Hospital's rule in the infinity - infinity case
        - Apply L'Hospital's rule in the infinity - infinity case with a sine or cosine function
        - Apply L'Hospital's rule in the infinity - infinity case with an exponential function
        - Apply L'Hospital's rule in the infinity - infinity case with a logarithmic function
    - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a power
        - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a power
        - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a power involving a sine or cosine function
        - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a power involving a logarithmic function
        - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a power involving an exponential function
    - Determine when a limit is an indeterminate form
- Growth Rates of Functions Using L'Hospital's Rule
    - Use L'Hospital's rule to compare the growth rate of two functions
        - Use L'Hospital's rule to compare the growth rate of two functions
        - Use L'Hospital's rule to compare the growth rate of two functions, one of which is an exponential function with base e
        - Use L'Hospital's rule to compare the growth rate of two functions, one of which is an exponential function with base other than e
        - Use L'Hospital's rule to compare the growth rate of two functions, one of which is a logarithmic function

4.9 Newton's Method

- Newton's Method
    - Set up an equation to find a value using Newton's method
        - Set up an equation to use Newton's method to find the root of a polynomial function
        - Set up an equation to use Newton's method to solve a trigonometric equation
        - Set up an equation to use Newton's method to solve an exponential equation
        - Set up an equation to use Newton's method to solve a logarithmic equation
    - Use Newton's method to approximate the root of a polynomial function or the solution of an algebraic equation
        - Find the root of a polynomial function using Newton's method
        - Solve a polynomial equation using Newton's method

        - Solve a rational equation using Newton's method
        - Solve a radical equation using Newton's method
    - Use Newton's method to approximate the value of a radical expression
        - Set up an equation to approximate the value of a radical expression using Newton's method
        - Use Newton's method to approximate the value of a radical expression
    - Use Newton's method to approximate the solution of a trigonometric or exponential equation
        - Solve an equation involving sine or cosine using Newton's method
        - Solve an equation involving an exponential function using Newton's method
        - Solve an equation involving a logarithmic function using Newton's method
    - Determine when Newton's method does not work
    - Use Newton's method in application
        - Use Newton's method to find the location of local extrema for polynomial functions
        - Use Newton's method with Kepler's equation regarding planetary orbits

4.10 Antiderivatives

- The Antiderivative
    - Find the general antiderivative of an algebraic function
        - Find the general antiderivative of a polynomial function
        - Find the general antiderivative of an algebraic function containing negative exponents
        - Find the general antiderivative of an algebraic function containing rational exponents
    - Find the general antiderivative of a trigonometric function
        - Find the general antiderivative of a sine or cosine function
        - Find the general antiderivative of other trigonometric functions
    - Find the general antiderivative of an exponential or logarithmic function
        - Find the general antiderivative of an exponential function
        - Find the general antiderivative of a logarithmic function
    - Find the general antiderivative of an inverse trigonometric function
- The Indefinite Integral
    - Understand integral notation and verify an indefinite integral
        - Verify an indefinite integral involving a polynomial
        - Verify an indefinite integral involving a trigonometric expression
        - Verify an indefinite integral involving an exponential expression
    - Understand the properties of indefinite integrals
    - Evaluate indefinite integrals involving algebraic functions
        - Evaluate an indefinite integral of a polynomial expression

        - Evaluate an indefinite integral of an algebraic expression containing negative exponents
        - Evaluate an indefinite integral of an algebraic expression containing rational exponents
    - Evaluate indefinite integrals involving trigonometric functions
        - Evaluate an indefinite integral of a sine or cosine function
        - Evaluate an indefinite integral of other trigonometric function
    - Evaluate indefinite integrals involving exponential or logarithmic function
        - Evaluate an indefinite integral of an exponential function
        - Evaluate an indefinite integral of a logarithmic function
    - Evaluating indefinite integrals involving inverse trigonometric functions
- Initial-Value Problems and Motion
    - Solve an initial-value problem
        - Solve an initial value problem for a polynomial function
        - Solve an initial value problem with roots or rational expressions
        - Solve an initial value problem for a trigonometric function involving sine or cosine
        - Solve an initial value problem for trigonometric functions other than sine or cosine
        - Solve an initial value problem given a second derivative and initial values for the first derivative and the function
        - Solve an initial value problem given a second derivative and two initial values for the function
    - Solve a word problem involving antiderivatives and motion
        - Find a position function given a velocity function and initial position
        - Find a position function given an acceleration function and an initial velocity and an initial position
        - Solve a word problem by finding a position function given initial conditions
        - Find the maximum height of a projectile
        - Find the time a projectile is in the air

Chapter 5: Integration

5.1 Approximating Areas

- Sigma Notation
    - Write a series of numbers using sigma notation and use the properties of sigma notation
        - Write a series of numbers using sigma notation
        - Evaluate a sum given in sigma notation
        - Use the properties of sigma notation
    - Use sigma notation to calculate sums and powers of integers
- Approximating Area

- Approximate the area under a curve using rectangles
    - Use rectangles to graphically represent a left-endpoint approximation of the area under a curve
    - Use rectangles to graphically represent a right-endpoint approximation of the area under a curve
    - Approximate the area under a curve using a left-endpoint approximation given a graph
    - Approximate the area under a curve using a right-endpoint approximation given a graph
    - Approximate the area under a curve using a left-endpoint approximation given a function
    - Approximate the area under a curve using a right-endpoint approximation given a function
- Approximate the area under a curve using midpoint approximation
    - Use rectangles to graphically represent a midpoint approximation of the area under a curve
    - Approximate the area under a curve using a midpoint approximation given a graph
    - Approximate the area under a curve using a midpoint approximation given a function
- Find an upper or lower sum given a function
    - Find the lower sum given a function
    - Find the upper sum given a function
    - Graph a visualization of an upper or lower sum
- Determine if a sum will be an underestimate or overestimate
- Solve application problems involving approximating areas

5.2 The Definite Integral

- Definition and Notation of Definite Integrals
    - Identify notation of a definite integral and describe when a function is integrable
    - Express a limit or a Riemann sum as a definite integral
        - Express a limit as a definite integral
        - Express a Riemann sum as a definite integral
    - Evaluate an integral using the definition of the definite integral and left- or right-endpoint approximations
        - Express a definite integral using a Riemann sum
        - Evaluate a definite integral using a Riemann sum
- Area and the Definite Integral
    - Use a geometric formula to calculate a definite integral

        - Use a geometric formula to calculate a definite integral of a linear function given a graph
        - Use a geometric formula to calculate a definite integral of an absolute value function given a graph
        - Use a geometric formula to calculate a definite integral of a radical function given a graph
        - Use a geometric formula to calculate a definite integral of a piecewise function given a graph
    - Calculate net signed areas under a function using geometric formulas
        - Use a geometric formula to find the net signed area of a linear function given a graph
        - Use a geometric formula to find the net signed area of an absolute value function given a graph
        - Use a geometric formula to find the net signed area of a piecewise function given a graph
        - Use a geometric formula to find the net signed area of a linear function
        - Use a geometric formula to find the net signed area of an absolute value function
    - Calculate total area under a function using geometric formulas
        - Use a geometric formula to find the total area of a linear function given a graph
        - Use a geometric formula to find the total area of an absolute value function given a graph
        - Use a geometric formula to find the total area of a piecewise function given a graph
        - Use a geometric formula to find the total area of a linear function
        - Use a geometric formula to find the total area of an absolute value function
- Properties of the Definite Integral and the Average Value of a Function Geometrically
    - Use the properties of the definite integral
    - Understand the comparison theorem of integrals and use it to compare two functions over a given interval
    - Calculate the average value of a function using a geometric argument
        - Find the average value of a function given the value of an integral
        - Calculate the average value of a function given a graph
        - Calculate the average value of a linear function using a geometric argument
        - Calculate the average value of a radical function using a geometric argument
    - Use the Mean Value Theorem for Integrals and a geometric argument to find where a function takes on its mean value
        - Find the point on a function where the function takes on its mean value
        - Use the Mean Value Theorem for Integrals to find where a linear function takes on its average value

Knewton Alta

5.3 The Fundamental Theorem of Calculus

- Fundamental Theorem of Calculus Part 1 - Integrals and Antiderivatives
  - Use the Fundamental Theorem of Calculus to find the derivative of an integral function
    - Use the Fundamental Theorem of Calculus to find the derivative of an integral function
    - Use the Fundamental Theorem of Calculus and a property of integrals to find the derivative of an integral function
    - Use the Fundamental Theorem of Calculus to find the derivative of an integral trigonometric function
    - Use the Fundamental Theorem of Calculus to find the derivative of an integral exponential or logarithmic function
  - Use the Fundamental Theorem of Calculus and the chain rule to find a derivative of an integral function
    - Use the Fundamental Theorem of Calculus and the chain rule to find a derivative of an integral function
    - Use the Fundamental Theorem of Calculus and the chain rule to find a derivative of an integral function with a radical bound
    - Use the Fundamental Theorem of Calculus and the chain rule to find a derivative of an integral function with a trigonometric bound
    - Use the Fundamental Theorem of Calculus and the chain rule to find a derivative of an integral function with an exponential or logarithmic bound
  - Use the Fundamental Theorem of Calculus with two variable limits of integration
    - Use the Fundamental Theorem of Calculus with two variable limits of integration
    - Use the Fundamental Theorem of Calculus with two variable limits of integration and a radical bound
    - Use the Fundamental Theorem of Calculus with two variable limits of integration and a trigonometric bound
    - Use the Fundamental Theorem of Calculus with two variable limits of integration and an exponential or logarithmic bound
- Fundamental Theorem of Calculus Part 2 - The Evaluation Theorem
  - Understand the Fundamental Theorem of Calculus graphically
    - Use the Fundamental Theorem of Calculus to relate a function defined by an integral and area under a curve
    - Use the Fundamental Theorem of Calculus to relate a function defined by an integral and its derivative
    - Use the Fundamental Theorem of Calculus to relate a function defined by an integral and its behavior
  - Evaluate definite integrals involving algebraic functions using the Fundamental Theorem of Calculus

        - Evaluate a definite integral of a polynomial expression
        - Evaluate a definite integral of an algebraic expression containing negative exponents
        - Evaluate a definite integral of an algebraic expression containing rational exponents
    - Evaluate definite integrals involving trigonometric functions
        - Evaluate a definite integral of a sine or cosine function
        - Evaluate a definite integral of other trigonometric functions
- Average Value of a Function with the Fundamental Theorem of Calculus
    - Calculate the average value of a function
        - Calculate the average value of a polynomial function
        - Calculate the average value of an algebraic expression containing negative exponents
        - Calculate the average value of an algebraic expression containing rational exponents
        - Calculate the average value of a sine or cosine function
        - Calculate the average value of other trigonometric functions
    - Use the Mean Value Theorem for Integrals to find the point on the curve which takes on the average value of the function

5.4 Net Change Theorem

- Introduction to the Net Change Theorem and Displacement Problems
    - Use the net change theorem to solve problems given graphs
        - Use the net change theorem given a graph of a function
        - Use the net change theorem given a graph of a derivative function
    - Find the net displacement of a particle given its velocity function
        - Find the net displacement of a particle given its velocity function
        - Find the net displacement of a particle given a velocity function with negative exponents
        - Find the net displacement of a particle given a velocity function with rational exponents
        - Find the net displacement of a particle given a velocity function with a sine or cosine function
    - Find the total distance traveled by a particle given its velocity function
- The Net Change Theorem in Other Contexts
    - Use the net change theorem to find amounts given rates of change in a scientific context
        - Use the net change theorem to find a change in volume
        - Use the net change theorem to find a population change
        - Use the net change theorem to find the net charge through a wire

        - Use the net change theorem to find the mass of a rod given its linear density
    - Use the net change theorem to find amounts given rates of change in a business or social science context
        - Use the net change theorem to find a change in cost given the marginal cost function
        - Use the net change theorem to find a change in revenue given the marginal revenue function
    - Use the net change theorem to find amounts given rates of change in a geometric context
        - Use the net change theorem to find amounts given rates of change in a geometric context
- Integrating Even and Odd Functions
    - Integrate an even or odd function
        - Find a definite integral using the properties of even functions
        - Find a definite integral using the properties of odd functions

5.5 Substitution

- U-Substitution and Indefinite Integrals
    - Use substitution to find an indefinite integral with the power rule and u given
        - Use substitution to find an indefinite integral with the power rule and u given
        - Use substitution to find an indefinite integral with the power rule and u given but with further manipulation needed
        - Use substitution to find an indefinite integral with the power rule for negative exponents and u given
        - Use substitution to find an indefinite integral with the power rule for negative exponents and u given but with further manipulation needed
        - Use substitution to find an indefinite integral with the power rule for rational exponents and u given
        - Use substitution to find an indefinite integral with the power rule for rational exponents and u given but with further manipulation needed
    - Use substitution to evaluate an indefinite integral with the power rule
        - Use substitution to evaluate an indefinite integral with the power rule
        - Use substitution to evaluate an indefinite integral with the power rule where further manipulation may be necessary
        - Use substitution to find an indefinite integral with the power rule for negative exponents
        - Use substitution to find an indefinite integral with the power rule for rational exponents
    - Use substitution to find an indefinite integral with trigonometric functions
        - Use substitution to find an indefinite integral with sine or cosine functions

        - Use substitution to find an indefinite integral with other trigonometric functions
- U-Substitution and Definite Integrals
    - Use substitution to evaluate a definite integral with the power rule
        - Determine the new limits of integration when evaluating a definite integral using substitution
        - Use substitution to evaluate a definite integral with the power rule
        - Use substitution to evaluate a definite integral with the power rule and negative exponents
        - Use substitution to evaluate a definite integral with the power rule and rational exponents
    - Use substitution to evaluate a definite integral with trigonometric functions
        - Determine the new limits of integration when evaluating a definite integral using substitution for trigonometric functions
        - Use substitution to evaluate a definite integral with sine or cosine functions
        - Use substitution to evaluate a definite integral with other trigonometric functions

5.6 Integrals Involving Exponential and Logarithmic Functions

- Indefinite Integrals with Exponential and Logarithmic Functions
    - Use substitution to find an indefinite integral with exponential functions
        - Use substitution to find an indefinite integral with exponential functions of base e
        - Use substitution to find an indefinite integral with exponential functions of a base other than e
        - Use substitution to find an indefinite integral with exponential functions of base e and negative exponents
        - Use substitution to find an indefinite integral with exponential functions of base e and rational exponents
    - Use substitution to find an indefinite integral involving exponential functions and trigonometric functions
        - Use substitution to find an indefinite integral involving exponential functions and sine or cosine functions
        - Use substitution to find an indefinite integral involving exponential functions and other trigonometric functions
    - Use substitution to find an indefinite integral involving logarithmic functions
        - Use substitution to find an indefinite integral of a rational function
        - Use substitution to find an indefinite integral involving natural logarithmic functions
        - Find an indefinite integral of a natural logarithmic function
        - Find an indefinite integral of a logarithmic function of another base
    - Use substitution to find an indefinite integral involving logarithmic functions and trigonometric functions

        - Use substitution to find an indefinite integral of a tangent function
        - Use substitution to find an indefinite integral involving natural logarithm and trigonometric functions
- Definite Integrals with Exponential and Logarithmic Functions and Applications
    - Use substitution to evaluate a definite integral with exponential functions
    - Use substitution to evaluate a definite integral with rational or logarithmic functions
        - Use substitution to evaluate a definite integral involving logarithmic functions
        - Use substitution to evaluate a definite integral involving logarithmic functions and trigonometric functions
    - Solve application problems using integrals of exponential functions
        - Solve an application involving population with base e
        - Find a price-demand equation given the marginal price-demand function with base e
        - Solve an application involving population with base other than e
    - Solve application problems using integrals involving logarithmic functions
        - Solve an application problem involving a rational function
        - Solve an application problem involving a natural logarithmic function
        - Solve an application problem involving a logarithmic function of another base

5.7 Integrals Resulting in Inverse Trigonometric Functions

- Integrals with Inverse Trigonometric Functions
    - Use substitution to find an indefinite integral involving an inverse trigonometric function
        - Use substitution to find an indefinite integral resulting in an inverse trigonometric function
        - Use substitution to find an indefinite integral involving an inverse trigonometric function
    - Use substitution to evaluate a definite integral involving an inverse trigonometric function

Chapter 6: Applications of Integration

6.1 Areas Between Curves

- Area Between Curves
    - Determine the area of a region between two curves defined by algebraic functions
        - Represent the area between two curves graphically
        - Write an integral to represent the area between two curves
        - Find the area of a region between two linear functions
        - Find the area of a region between two functions with a polynomial
        - Find the area of a region between two functions with rational exponents
        - Find the area of a region between two functions with negative exponents
    - Determine the area of a region between two curves involving transcendental functions

        - Find the area of a region between two curves with a sine or cosine function
        - Find the area of a region between two curves with an exponential function
        - Find the area of a region between two curves where integrating results in a natural logarithm
- Area of Compound Regions
    - Determine the area of a compound region defined by algebraic functions
        - Find the area bounded by two functions that intersect
        - Find the area bounded above or below by two different functions
        - Find the area bounded above or below by a radical function and another function
        - Find the area bounded above or below by a rational function and another function
    - Determine the area of a compound region involving transcendental functions
        - Find the area bounded by two trigonometric functions that cross
        - Find the area bounded above or below by an exponential function and another function
        - Find the area bounded above or below by functions where integrating results in a natural logarithmic function
- Area Between Curves for Functions of y and Applications
    - Determine the area of a region between two curves defined by algebraic functions by integrating with respect to the dependent variable
        - Write an integral to represent the area between two curves defined as functions of y
        - Find the area between two linear functions defined as functions of y
        - Find the area between two functions defined as functions of y
        - Find the area between two functions defined as functions of y with rational exponents
    - Determine the area of a region between two curves by rewriting functions in terms of y
        - Rewrite functions as functions of y to make a compound region a single integral
        - Find the area between two curves by rewriting functions as functions of y
        - Find the area between two curves by rewriting functions as functions of y with rational exponents
    - Use the area between curves to solve application problems
        - Find the total profit given marginal cost and marginal revenue functions
        - Find the distance between objects given two velocity functions
        - Find a total accumulation given two rate of change functions

6.2 Determining Volumes by Slicing

- Determine Volumes Using Cross-Sections
    - Determine the volume of a solid by integrating a cross-section

        - Find the volume of a solid with a base formed by two functions and square cross sections
        - Find the volume of a solid with a base formed by two functions and equilateral triangle cross sections
        - Find the volume of a solid with a base formed by two functions and semicircle cross sections
        - Find the volume of a solid with cross sections and a circular base
- Determine Volumes with the Disc Method
    - Find the volume of a solid of revolution using the disk method
        - Set up an integral to find the volume of a solid of revolution around the x-axis with the disc method
        - Use the disc method to find the volume of a solid of revolution with a polynomial function
        - Use the disc method to find the volume of a solid of revolution with a function with rational exponents
        - Use the disc method to find the volume of a solid of revolution with a sine or cosine function
        - Use the disc method to find the volume of a solid of revolution with an exponential function
        - Set up an integral to find the volume of a solid of revolution around the y-axis with the disc method
- Determine Volumes with the Washer Method
    - Find the volume of a solid of revolution with a cavity using the washer method
        - Set up an integral to find the volume of a solid of revolution using the washer method
        - Use the washer method to find the volume of a solid of revolution with a polynomial function
        - Use the washer method to find the volume of a solid of revolution with a function with rational exponents
        - Use the washer method to find the volume of a solid of revolution with an exponential function
        - Set up an integral to find the volume of a solid of revolution around a line other than the x-axis using the washer method
    - Find the volume of a solid of revolution using the washer method for functions of y
        - Set up an integral to find the volume of a solid of revolution using the washer method for a function of y
        - Use the washer method to find the volume of a solid of revolution with a polynomial function in terms of y

        - Use the washer method to find the volume of a solid of revolution with a function in terms of y with rational exponents
        - Set up an integral to find the volume of a solid of revolution around a line other than the y-axis using the washer method for functions of y
    - Find the volume of a solid of revolution using the washer method by rewriting a function as a function of y
        - Set up an integral to find the volume of a solid of revolution using the washer method by rewriting a function as a function of y
        - Use the washer method to find the volume of a solid of revolution by rewriting a function in terms of y
        - Use the washer method to find the volume of a solid of revolution by rewriting a function in terms of y with rational exponents
        - Set up an integral to find the volume of a solid of revolution around a line other than the y-axis using the washer method and rewriting functions in terms of y

6.3 Volumes of Revolution - Cylindrical Shells

- Determine Volumes with the Shell Method
    - Calculate the volume of a solid of revolution by using the method of cylindrical shells
        - Set up an integral to find the volume of a solid of revolution using the shell method
        - Use the shell method to find the volume of a solid of revolution with a polynomial function
        - Use the shell method to find the volume of a solid of revolution with a function with rational exponents
        - Use the shell method to find the volume of a solid of revolution with a sine or cosine function
        - Use the shell method to find the volume of a solid of revolution with an exponential function
        - Set up an integral to find the volume of a solid of revolution around a line other than the y-axis using the shell method
    - Calculate the volume of a solid of revolution by using the method of cylindrical shells for functions of y
        - Set up an integral to find the volume of a solid of revolution using the shell method for a function of y
        - Use the shell method to find the volume of a solid of revolution with a polynomial function in terms of y
        - Use the shell method to find the volume of a solid of revolution with a function in terms of y with rational exponents
        - Set up an integral to find the volume of a solid of revolution around a line other than the x-axis using the shell method for functions of y

- Compare Methods for Volumes of Revolution
    - Choose an appropriate method to find the volume of a solid of revolution

6.4 Arc Length of a Curve and Surface Area

- Calculating Arc Length
    - Determine the length of a curve f(x) between two points
        - Set up an integral to find an arc length
        - Find the arc length of a linear function
        - Find the arc length of a function with rational exponents
        - Find the arc length of a function with negative exponents
        - Use a calculator to find the arc length of a function
    - Determine the length of a curve g(y) between two points
        - Set up an integral to find the arc length of a curve of g(y)
        - Find the arc length of a linear function given as g(y)
        - Find the arc length of a function with negative exponents given as g(y)
        - Use a calculator to find the arc length of a function given as g(y)
        - Rewrite a function given as f(x) to g(y) to find arc length
- Area of a Surface of Revolution
    - Find the surface area of a surface of revolution around the x-axis
        - Set up an integral to find the surface area from revolving a curve around the x-axis
        - Find the surface area of a revolution of a polynomial function around the x-axis
        - Find the surface area of a revolution of a function with rational exponents around the x-axis
        - Use a calculator to find the surface area of a surface of revolution around the x-axis
    - Find the surface area of a surface of revolution with a function g(y)
        - Set up an integral to find the surface area from revolving a curve around the y-axis
        - Find the surface area of a revolution of a polynomial function around the y-axis
        - Use a calculator to find the surface area of a surface of revolution around the y-axis
        - Rewrite a function given as f(x) to g(y) to find surface area

6.5 Physical Applications

- Mass and Density
    - Determine the mass of a one-dimensional object from its linear density function
        - Determine the mass of a one-dimensional object given a polynomial linear density function
        - Determine the mass of a one-dimensional object given a linear density function with rational exponents

        - Determine the mass of a one-dimensional object given a linear density function with negative exponents
        - Determine the mass of a one-dimensional object from a sine or cosine linear density function
        - Determine the mass of a one-dimensional object from an exponential linear density function
    - Determine the mass of a two-dimensional object from its radial density function
        - Determine the mass of a two-dimensional object given a polynomial radial density function
        - Determine the mass of a two-dimensional object given a radial density function with rational exponents
        - Determine the mass of a two-dimensional object given a radial density function with negative exponents
        - Determine the mass of a two-dimensional object given a sine or cosine radial density function
        - Determine the mass of a two-dimensional object given an exponential radial density function
- Work Done by a Force
    - Calculate the work done by a force acting along a line
        - Find the amount of work done given a constant force
        - Find the amount of work done to lift an object given its mass
        - Calculate the work done by a variable force acting along a line
    - Calculate the work done to compress or stretch a spring
        - Calculate the amount of work done to compress or stretch a spring given an initial force
        - Calculate the amount of work done to compress or stretch a spring given an initial work
        - Find the length of a spring given two initial amounts of work
    - Calculate the work done to lift a cable
        - Calculate the amount of work necessary to lift a cable
        - Calculate the amount of work necessary to lift a cable and an attached object
        - Calculate the amount of work necessary to lift a cable and an attached object that is losing mass
- Work Done in Pumping and Hydrostatic Force
    - Calculate the work done in pumping a liquid out of a tank
        - Calculate the work done in pumping a liquid out of a cylindrical tank
        - Calculate the work done in pumping a liquid out of a tank in the shape of an inverted cone
    - Find the hydrostatic force against a submerged plate

- Calculate the hydrostatic force on a submerged horizontal plate
- Calculate the hydrostatic force on a submerged vertical plate in the shape of an isosceles triangle
- Calculate the hydrostatic force on a submerged vertical plate in the shape of a trapezoid
- Calculate the hydrostatic force on a submerged vertical plate in the shape of a circle or semicircle

6.6 Moments and Centers of Mass

- Centers of Mass on a Line or Plane
  - Find the moment and center of mass of objects distributed along a line
    - Find the moment of objects distributed along a line
    - Find the center of mass of objects distributed along a line
    - Represent the center of mass of objects distributed along a line graphically
  - Find the moments and the center of mass of objects in a plane
    - Find the moments of objects in a plane
    - Find the center of mass of objects in a plane
    - Represent the center of mass of objects distributed in a plane graphically
- Centroids of Regions on a Plane
  - Find the mass, moments, and center of mass of a lamina with uniform density
    - Find the mass of a lamina
    - Find the moments of a lamina
    - Find the center of mass of a lamina bounded by a function using the mass and moments
    - Find the center of mass of a lamina bounded by two functions using the mass and moments
  - Find the centroid of a lamina of uniform density using the area
    - Find the centroid of a lamina of uniform density bounded by a function using the area
    - Find the centroid of a lamina bounded by two functions using the area
  - Use symmetry to help locate the centroid of a thin plate
  - Apply the theorem of Pappus for volume
    - Use the Theorem of Pappus to find the volume of a solid given coordinate points
    - Use the Theorem of Pappus to find the volume of a torus

6.7 Exponential Growth and Decay

- Exponential Growth Models
  - Use an exponential growth model to solve problems involving population
    - Find a population given a function
    - Find when a population will reach a given number given a function
    - Write a function to model population growth

- Solve compound interest problems
  - Solve compound interest problems with discrete compounding intervals
  - Use continuously compounding interest to find a new balance
  - Use continuously compounding interest to find when an account will reach a given balance
  - Use continuously compounding interest to find the principal required to have a given balance
- Use the concept of doubling time to solve exponential growth problems
  - Find the amount of time necessary for an initial value to reach a given multiple
  - Use the concept of doubling time to find a growth rate
  - Use the concept of doubling time to solve exponential growth problems

- Exponential Decay Models
  - Use an exponential decay model to solve problems involving Newton's Law of Cooling
    - Write a function to model temperature following Newton's Law of Cooling
    - Use Newton's Law of Cooling to find the temperature of an object
    - Use Newton's Law of Cooling to find when an object will reach a given temperature
  - Use the concept of half-life to solve radioactive decay problems
    - Write a function to model exponential decay in half-life problems
    - Determine the mass of a sample remaining in a half-life problem
    - Determine the age of a sample given a resultant mass in a half-life problem

6.8 Calculus of the Hyperbolic Functions

- Derivatives and Integrals of the Hyperbolic Functions
  - Find the derivative of a hyperbolic function
    - Find the derivative of a hyperbolic sine or cosine function
    - Find the derivative of a hyperbolic tangent or cotangent function
    - Find the derivative of a hyperbolic secant or cosecant function
  - Evaluate an integral involving a hyperbolic function
    - Evaluate an indefinite integral involving a hyperbolic sine or cosine function
    - Evaluate an indefinite integral involving a hyperbolic tangent or cotangent function
    - Evaluate an indefinite integral involving a hyperbolic secant or cosecant function
  - Use a catenary to find the length of a hanging cable
- Calculus of Inverse Hyperbolic Functions
  - Find the derivative of an inverse hyperbolic function
    - Find the derivative of an inverse hyperbolic sine or cosine function
    - Find the derivative of an inverse hyperbolic tangent or cotangent function
    - Find the derivative of an inverse hyperbolic secant or cosecant function
  - Evaluate an integral involving an inverse hyperbolic function

- Evaluate an indefinite integral involving an inverse hyperbolic sine or cosine function
- Evaluate an indefinite integral involving an inverse hyperbolic tangent or cotangent function
- Evaluate an indefinite integral involving an inverse hyperbolic secant or cosecant function

Chapter 7: Techniques of Integration

7.1 Integration by Parts

- Integration by Parts for Indefinite Integrals
  - Use integration by parts to find indefinite integrals given u and dv
    - Use the integration by parts formula to solve an indefinite integral with a logarithmic function, given u and dv
    - Use the integration by parts formula to solve an indefinite integral with an exponential function given u and dv
    - Use the integration by parts formula to solve an indefinite integral with a trigonometric function, given u and dv
  - Use integration by parts to find indefinite integrals
    - Use the integration by parts formula once to solve an indefinite integral with a logarithmic function
    - Use the integration by parts formula once to solve an indefinite integral with an exponential function
    - Use the integration by parts formula once to solve an indefinite integral with a trigonometric function
    - Use the integration by parts formula to solve an indefinite integral with a single function
  - Use integration by parts more than once to find indefinite integrals
    - Use the integration by parts formula more than once to solve an indefinite integral with a logarithmic function
    - Use the integration by parts formula more than once to solve an indefinite integral with an exponential function
    - Use the integration by parts formula more than once to solve an indefinite integral with a trigonometric function
    - Use the integration by parts formula to identify a pattern that solves an indefinite integral with a logarithmic function
    - Use the integration by parts formula to identify a pattern that solves an indefinite integral with an exponential function
- Integration by Parts for Definite Integrals and Applications
  - Use integration by parts for definite integrals

        - Use the integration by parts formula to evaluate definite integrals with logarithmic functions
        - Use the integration by parts formula to evaluate definite integrals with exponential functions
        - Use the integration by parts formula to evaluate definite integrals with trigonometric functions
    - Use integration by parts to solve area and volume problems
        - Use the integration by parts formula to find the area of a region with logarithmic or exponential functions
        - Use the integration by parts formula to find the area of a region with trigonometric functions
        - Use the integration by parts formula to find the volume of a solid with logarithmic or exponential functions
        - Use the integration by parts formula to find the volume of a solid with trigonometric functions

7.2 Trigonometric Integrals

- Integrating Products and Powers of Trigonometric Functions
    - Solve trigonometric integrals with products and powers of sine and cosine
        - Solve trigonometric integrals with products and powers of sine and cosine where all exponents are odd
        - Solve trigonometric integrals with products and powers of sine and cosine where one exponent is odd
        - Solve trigonometric integrals with products and powers of sine and cosine where all exponents are even
        - Solve definite integrals with products and powers of sine and cosine where all exponents are odd
        - Solve definite integrals with products and powers of sine and cosine where one exponent is odd
        - Solve definite integrals with products and powers of sine and cosine where all exponents are even
    - Solve trigonometric integrals with products and powers of tangent and secant
        - Solve trigonometric integrals with products and powers of tangent and secant where the secant function has an even power
        - Solve trigonometric integrals with products and powers of tangent and secant where all exponents are odd
        - Solve trigonometric integrals with products and powers of tangent
        - Solve trigonometric integrals with products and powers of secant using a reduction formula
        - Solve definite integrals with products and powers of tangent and secant

- Reduction Formulas
    - Solve trigonometric integrals with different angles using product-to-sum identities
        - Solve trigonometric integrals of one sine and one cosine function with different angles using product to sum identities
        - Solve trigonometric integrals of two sine functions with different angles using product to sum identities
        - Solve trigonometric integrals of two cosine functions with different angles using product to sum identities

7.3 Trigonometric Substitution

- Integration by Trigonometric Substitution
    - Solve integration problems involving a difference of squares of the form a^2-x^2 using a trigonometric substitution
        - Evaluate an indefinite integral containing a difference of squares of the form a^2-x^2
        - Evaluate a definite integral containing a difference of squares of the form a^2-x^2
    - Solve integration problems involving a sum of squares using a trigonometric substitution
        - Evaluate an indefinite integral containing a sum of squares
        - Evaluate a definite integral containing a sum of squares
        - Use a hyperbolic trigonometric substitution to evaluate an indefinite integral containing a sum of squares
    - Solve integration problems involving a difference of squares of the form x^2-a^2 using a trigonometric substitution
        - Evaluate an indefinite integral containing a difference of squares of the form x^2-a^2
        - Evaluate a definite integral containing a difference of squares of the form x^2-a^2
        - Use a hyperbolic trigonometric substitution to evaluate an indefinite integral containing a difference of squares of the form x^2-a^2
- Applications and Extensions of Integration by Trigonometric Substitution
    - Solve application problems involving trigonometric substitution
        - Use a trigonometric substitution to find an arc length
        - Find the area of a region under a curve using integration involving a trigonometric substitution
    - Solve integration problems requiring the process of completing the square
        - Use the techniques of completing the square and a sine substitution to evaluate an integral
        - Use the techniques of completing the square and a tangent substitution to evaluate an integral

- Use the techniques of completing the square and a secant substitution to evaluate an integral

7.4 Partial Fractions

- Integration Using Polynomial Long Division
  - Integrate a rational function using long division of polynomials
- Integration Using Partial Fractions
  - Integrate a rational function using partial fractions with nonrepeated linear factors
    - Integrate a rational function using partial fractions with nonrepeated linear factors
    - Integrate a rational function using long division and partial fractions with nonrepeated linear factors
    - Integrate a rational function using partial fractions with nonrepeated linear factors after a substitution
  - Integrate a rational function using partial fractions with repeated linear factors or a quadratic factor
    - Integrate a rational function using partial fractions with repeated linear factors
    - Integrate a rational function using partial fractions with a nonrepeated quadratic factor
    - Integrate a rational function using partial fractions with a repeated quadratic factor
  - Integrate a function using partial fractions in applications

7.5 Other Strategies for Integration

- Integration Using a Table of Integrals
  - Use a table of integrals to evaluate integral problems
    - Determine the appropriate formula to evaluate the integral
    - Evaluate an integral given an integral formula
    - Evaluate an integral given a table of integrals
    - Evaluate an integral given a table of integrals with u-substitution
- Integration Using a Computer Algebra System
  - Use a computer algebra system (CAS) to solve integral problems
    - Use a CAS to evaluate integrals with polynomials or rational functions
    - Use a CAS to evaluate integrals with trigonometric functions
    - Use Desmos to evaluate definite integrals
  - Use a computer algebra system (CAS) to solve application problems with integrals
    - Solve area application problems with a CAS
    - Solve area application problems with Desmos
    - Solve volume application problems with a CAS
    - Solve volume application problems with Desmos

Knewton Alta

7.6 Numerical Integration

- Trapezoidal Rule
  - Approximate a definite integral using the Trapezoid Rule
    - Approximate the area under a curve using the Trapezoid method given a graph
    - Approximate the area under a curve using the Trapezoid method given a function
    - Represent a trapezoid approximation of the area under a curve graphically
- Absolute and Relative Error
  - Determine the absolute and relative error in using the midpoint or trapezoidal rules
    - Calculate the absolute error in using a midpoint approximation
    - Calculate the absolute error in using a trapezoidal approximation
    - Calculate the relative error in using a midpoint approximation
    - Calculate the relative error in using a trapezoidal approximation
  - Determine error bounds for midpoint and trapezoidal rules
    - Calculate an upper bound for the error using the midpoint rule
    - Calculate an upper bound for the error using the Trapezoid rule
    - Determine the number of intervals necessary to reach a given level of accuracy using the midpoint rule
    - Determine the number of intervals necessary to reach a given level of accuracy using the trapezoid rule
- Simpson's Rule
  - Approximate a definite integral using Simpson's rule
    - Approximate the area under a curve using Simpson's rule given a graph
    - Approximate the area under a curve using Simpson's rule given a table
    - Approximate the area under a curve using Simpson’s rule given a function
  - Determine error and error bounds in using Simpson's rule
    - Calculate the absolute error in using Simpson's rule
    - Calculate the relative error in using Simpson's rule
    - Calculate an upper bound for the error using Simpson's rule
    - Determine the number of intervals necessary to reach a given level of accuracy using Simpson's rule

7.7 Improper Integrals

- Integrating over an Infinite Interval
  - Evaluate an improper integral over an infinite interval
    - Evaluate improper integrals that are continuous over an infinite interval
    - Evaluate improper integrals that are continuous over an infinite interval using L'Hospital's Rule
- Integrating a Discontinuous Integrand
  - Evaluate an improper integral with an infinite discontinuity

- Evaluate an improper integral with an infinite discontinuity at one endpoint
- Evaluate an improper integral of a trigonometric function with an infinite discontinuity at one endpoint
- Evaluate an improper integral with an infinite discontinuity within the limits of integration
- Evaluate an improper integral with an infinite discontinuity at one endpoint using L'Hospital's Rule

- The Comparison Theorem and Convergence
  - Use the Comparison Theorem to determine whether an improper integral is convergent
    - Determine whether an improper integral with rational functions is convergent using the Comparison Theorem
    - Demonstrate a conceptual understanding of the Comparison Theorem
    - Determine whether an improper integral with exponential functions is convergent using the Comparison Theorem
    - Determine whether an improper integral with logarithmic functions is convergent using the Comparison Theorem
    - Determine whether an improper integral with trigonometric functions is convergent using the Comparison Theorem

Chapter 8: Introduction to Differential Equations

8.1 Basics of Differential Equations

- General Solutions to Differential Equations
  - Identify basic elements of differential equations
    - Identify the order of a differential equation
    - Verify that a polynomial or rational function is a general solution of a differential equation
    - Verify that a trigonometric function is a general solution of a differential equation
    - Verify that an exponential function is a general solution of a differential equation
    - Verify that a logarithmic function is a general solution of a differential equation
  - Solve general solution problems for linear differential equations
    - Find a general solution to a differential equation with polynomials using integration
    - Find a general solution to a differential equation with trigonometric functions using integration
    - Find a general solution to a differential equation with exponential functions using integration
    - Find a general solution to a differential equation with logarithmic functions using integration
- Initial Value Problems

- Verify a solution to an initial value problem of a linear differential equation
    - Verify a solution to a differential equation initial value problem with polynomials
    - Verify a solution to a differential equation initial value problem with trigonometric functions
    - Verify a solution to a differential equation initial value problem with exponential functions
    - Verify a solution to a differential equation initial value problem with logarithmic functions
- Find a solution to a linear differential equation initial value problem
    - Find a solution to a linear differential equation initial value problem with polynomials
    - Find a solution to a linear differential equation initial value problem with trigonometric functions
    - Find a solution to a linear differential equation initial value problem with exponential functions
    - Find a solution to a linear differential equation initial value problem with logarithmic functions
    - Identify a particular solution on a graph that satisfies an initial value problem
- Solve applications of linear differential equations
    - Solve motion applications with velocity using a differential equation initial value problem
    - Solve motion applications with position using a differential equation initial value problem

8.2 Direction Fields and Numerical Methods

- Create and Use a Direction Field
    - Introduction to direction fields
        - Find the slope of a solution curve at a given point
        - Plot the slope of a solution curve at a given point using technology
    - Use technology to sketch direction fields for a given differential equation
    - Use technology to sketch solution curves for a given initial-value problem
- Equilibrium Solutions
    - Identify and classify equilibrium solutions using a direction field
        - Identify an equilibrium solution given a direction field
        - Classify an equilibrium solution as stable, unstable, or semi-stable
- Euler's Method
    - Use Euler’s Method to approximate the solution to a first-order differential equation
        - Estimate the solution to a differential equation involving a polynomial expression using Euler's Method

        - Estimate the solution to a differential equation involving an exponential expression using Euler's Method
        - Estimate the solution to a differential equation involving a trigonometric expression using Euler's Method
    - Estimate the error in Euler's Method
        - Estimate the error in Euler's Method when applied to a differential equation involving a polynomial expression
        - Estimate the error in Euler's Method when applied to a differential equation involving an exponential expression
        - Estimate the error in Euler's Method when applied to a differential equation involving a trigonomic expression

8.3 Separable Equations

- Finding General Solutions Using Separation of Variables
    - Use the separation of variables technique to find constant solutions and set up integrals to solve differential equations
        - Find any constant solutions for differential equations
        - Use the separation of variables technique to set up the integral that will solve a differential equation
        - Use the separation of variables technique to set up the integral that will solve a differential equation when factoring is required
    - Find general solutions to differential equations using separation of variables
        - Find general solutions of differential equations with rational functions
        - Find general solutions of differential equations with polynomial functions using separation of variables
        - Find general solutions of differential equations with polynomial functions using separation of variables when factoring is required
        - Find general solutions of differential equations with trigonometric functions using separation of variables
        - Find general solutions of differential equations with exponential functions using separation of variables
        - Find general solutions of differential equations with logarithmic functions using separation of variables
- Finding Particular Solutions and Solving Applications Using Separation of Variables
    - Find particular solutions to differential equations using separation of variables
        - Find particular solutions of differential equations with rational functions
        - Find particular solutions of differential equations with polynomial functions using separation of variables
        - Find particular solutions of differential equations with trigonometric functions using separation of variables

        - Find particular solutions of differential equations with exponential functions using separation of variables
        - Find particular solutions of differential equations with logarithmic functions using separation of variables
    - Solve differential equations application problems using separation of variables
        - Solve Newton's Law of Cooling problems with separation of variables
        - Solve solution concentration problems with separation of variables

8.4 The Logistic Equation

- Population Growth and Carrying Capacity
    - Model population growth using a logistic differential equation given information about the population
        - Identify the carrying capacity and growth rate of a given logistic equation
        - Identify the values for which a logistic equation is increasing or decreasing
        - Write the equation for a population problem using the logistic equation
    - Use a logistic equation to predict population size
        - Find the solution to a logistic differential equation
        - Use the solution to a logistic differential equation to predict population size
        - Find an equation to model population given initial information
- Solving the Logistic Differential Equation
    - Sketch a direction field for the logistic equation
        - Draw a direction field for the logistic equation when given the growth rate and carrying capacity
        - Draw the solution curve for a logistic equation for a given initial population
    - Interpret the solution of a logistic equation
        - Use solutions of initial value problems to find population sizes
        - Use solutions of initial value problems to find the time it takes to reach a population level
        - Use the solution of a logistic equation to determine how long it will take to reach the carrying capacity
- Other Models of Population Growth
    - Using other models of population growth
        - Use the natural growth model
        - Use the Gompertz growth function
        - Use logistic models where the population becomes extinct
        - Use logistic models where the population is harvested

8.5 First-Order Linear Equations

- Writing a First-Order Linear Differential Equation in Standard Form
    - Determine whether a first-order differential equation is linear
        - Determine whether a differential equation with polynomial functions is linear

        - Determine whether a differential equation with trigonometric functions is linear
        - Determine whether a differential equation with logarithmic functions is linear
    - Write a first-order linear differential equation in standard form
        - Rewrite a first-order differential equation with polynomial functions in standard form
        - Rewrite a first-order differential equation with trigonometric functions in standard form
        - Rewrite a first-order differential equation with logarithmic functions in standard form
        - Identify P(x) and Q(x) with polynomial functions
        - Identify P(x) and Q(x) with trigonometric functions
        - Identify P(x) and Q(x) with logarithmic functions
- Integrating Factors
    - Find an integrating factor
        - Find an integrating factor with polynomial functions
        - Find an integrating factor with trigonometric functions
        - Find an integrating factor with logarithmic functions
    - Use an integrating factor to solve first-order linear differential equations
        - Find the general solution to a first-order differential equation when the integrating factor is logarithmic
        - Find the general solution to a first-order differential equation when the integrating factor is trigonometric
        - Find the general solution to a first-order differential equation when the integrating factor is a polynomial
        - Find the solution to an initial-value first order differential equation when the integrating factor is a logarithmic
        - Find the solution to an initial-value first order differential equation when the integrating factor is trigonometric
        - Find the solution to an initial-value first order differential equation when the integrating factor is a polynomial
- Applications of First-Order Linear Differential Equations
    - Solve applications of first-order linear differential equations
        - Solve first-order differential equation problems related to electric circuits
        - Solve first-order differential equation problems related to volume mixing
        - Solve first-order differential equation problems related to free fall

Chapter 9: Sequences and Series

9.1 Sequences

- Finding Formulas for Sequences

- Write the terms of a sequence
  - Write the terms of an arithmetic sequence defined explicitly
  - Write the terms of a geometric sequence defined explicitly
  - Write the terms of a sequence defined explicitly
  - Write the terms of an arithmetic sequence defined recursively
  - Write the terms of a geometric sequence defined recursively
  - Write the terms of a sequence defined recursively
- Find the formula for the nth term of a sequence
  - Identify the explicit formula for the nth term of an infinite sequence
  - Identify the explicit formula for the nth term of an alternating infinite sequence
  - Write the explicit formula for the nth term of an infinite sequence
  - Write the explicit formula for the nth term of an alternating infinite sequence
  - Write the explicit formula for the nth term of a recursively defined sequence
- Determining the Limit of a Sequence
  - Determine the limit of a sequence graphically
  - Determine the limit of a sequence
    - Determine the limit of a sequence
    - Determine the limit of a sequence defined by a rational function
    - Determine the limit of a sequence defined by an exponential function
    - Determine the limit of a sequence defined by a trigonometric function
- Determining the Convergence of a Sequence
  - Determine whether a sequence converges or diverges
  - Determine whether a continuous function converges or diverges
    - Determine if a continuous function defined on a convergent sequence converges and if so find its limit
    - Determine if a sequence converges or diverges using L'Hospital's Rule
    - Determine the limit of a convergent sequence using the Squeeze Theorem
  - Use properties of bounded and monotonic sequences to determine convergence
    - Determine if a sequence is monotonic
    - Determine if a sequence is bounded
    - Determine convergence of a sequence using the Monotone Convergence Theorem

9.2 Infinite Series

- Determining Convergence of a Series
  - Determine whether a series converges or diverges
    - Find a partial sum of a given series
    - Determine if a series converges or diverges given the sequence of partial sums
    - Determine if a series converges or diverges using a sequence of partial sums
    - Determine the sum of a convergent series using algebraic properties

- Determine convergence of a geometric series
    - Determine if a series is geometric
    - Determine the common ratio of a geometric series
    - Determine if a geometric series converges
    - Determine the sum of a convergent geometric series
- Telescoping Series
    - Determine the convergence of a telescoping series
        - Determine if a series is telescoping
        - Determine if a telescoping series converges or diverges
        - Determine the sum of a telescoping series

9.3 The Divergence and Integral Tests

- Divergence and Integral Tests
    - Determine if a series diverges using the divergence test
        - Calculate an infinite limit
        - Calculate an infinite limit with a factorial expression
        - Use the divergence test for series
        - Use the divergence test for series with a factorial expression
    - Determine if a series converges or diverges using the integral test
        - Determine whether the integral test can be used for a given series
        - Determine the integral that can be used to represent a series for the integral test
        - Determine whether a series is convergent using the integral test
- The p-Series and Estimating Values of Series
    - Determine if a p-series converges or diverges
        - Identify the value of p for a given series
        - Determine whether a p-series is convergent
    - Estimate the value of a convergent series
        - Estimate the error for a given partial sum of a convergent series
        - Calculate the value of n that guarantees the remainder is within a certain error bound
        - Use a CAS to calculate a partial sum for a convergent series
        - Calculate the lower and upper bounds of a convergent series

9.4 Comparison Tests

- Comparison Tests
    - Use the comparison test to determine if a series converges or diverges
        - Select an appropriate series to use with the comparison test to show that a given series converges
        - Select an appropriate series to use with the comparison test to show that a given series diverges

        - Use the comparison test to determine whether the given infinite series converges or diverges
    - Use the limit comparison test to determine if a series converges or diverges
        - Calculate the infinite limit of a ratio of two series
        - Use the limit comparison test and a given geometric series to determine whether an infinite series converges or diverges
        - Use the limit comparison test to determine if a given infinite series converges or diverges
    - Use a comparison test to determine an unknown value for which a series converges

9.5 Alternating Series

- Alternating Series
    - Determine whether an alternating series converges
        - Determine convergence of an infinite series using the Alternating Series Test for a series written as a partial sum
        - Determine convergence of an infinite series using the Alternating Series Test for a series written in summation notation
        - Determine the value of an unknown quantity in a series that makes the series converge
        - Determine the value of an unknown quantity in a series that makes the series diverge
    - Approximate the sum of an alternating series
        - Estimate the remainder of an alternating series
        - Use the alternating series remainder to approximate the sum of an alternating series
        - Use the alternating series remainder to find the number of terms required to approximate the sum of a series
    - Understand absolute convergence and conditional convergence
        - Identify a convergent series as having absolute convergence or conditional convergence
        - Determine whether a series is divergent, absolutely convergent, or conditionally convergent

9.6 Ratio and Root Tests

- Ratio and Root Tests
    - Determine absolute convergence of a series using the Ratio Test
    - Determine absolute convergence of a series using the Root Test
    - Determine the appropriate convergence test for a series
        - Identify whether a series is geometric, harmonic, p, or alternating
        - Identify the test that is appropriate for determining the convergence of a series

Knewton Alta

Chapter 10: Power Series

10.1 Power Series and Functions

- Power Series and Functions
  - Identify a power series
  - Determine the interval of convergence and radius of convergence of a power series
    - Determine the interval of convergence of a power series that has passed the ratio test
    - Determine the interval of convergence of a power series
    - Determine the radius of convergence of a power series
  - Represent a function using a power series
    - Use a power series to represent a function
    - Use a power series to represent a function not centered at 0
    - Determine the interval of convergence for functions represented by power series
    - Determine the interval of convergence for functions represented by power series not centered at 0

10.2 Properties of Power Series

- Combining Power Series
  - Combine power series
    - Combine power series by addition or subtraction
    - Determine the interval of convergence for a series combined by addition or subtraction
    - Multiply two power series together
    - Determine the interval of convergence of a series created by multiplying two power series
  - Create a new power series
    - Create a power series from a known power series
    - Create a power series from a known power series using partial fractions
    - Determine the interval convergence of a series that has been created by a combination of two series
    - Find the function represented by a given power series
    - Determine the interval of convergence of the function represented by a given power series
- Differentiating and Integrating Power Series
  - Differentiate power series
    - Differentiate power series term-by-term
    - Determine convergence after differentiation
    - Evaluate the sum of a series
  - Integrate power series
    - Integrate power series term-by-term

- Determine convergence after integration

10.3 Taylor and Maclaurin Series

- Taylor and Maclaurin Series
  - Find a Taylor or Maclaurin polynomial for a function
    - Recognize if a given series is a Taylor series
    - Find a Taylor polynomial for a function at a given value
    - Find a Maclaurin polynomial for a function
  - Estimate a function value using a Taylor or Maclaurin polynomial
    - Estimate a function value using a Taylor polynomial
    - Estimate a function value using a Maclaurin polynomial
- Taylor's Theorem with Remainder
  - Use Taylor's Theorem with Remainder to answer questions regarding the error of a Taylor or Maclaurin series
    - Verify that a given degree of a Taylor polynomial would result in a desired accuracy
    - Determine the error of an estimated function value using Taylor's Theorem with Remainder
    - Determine the degree of a Maclaurin polynomial necessary for a desired accuracy
    - Determine the interval of convergence of a Taylor or Maclaurin series

10.4 Working with Taylor Series

- Binomial Series
  - Use a binomial series
    - Find the binomial series for a function
    - Estimate the value of a function using a binomial series
    - Use a binomial expansion to find the Taylor series of a function given a center
- Functions Expressed as Taylor Series
  - Find the Taylor expansion of common functions
    - Find the Taylor series for an exponential function
    - Find the Taylor series for a logarithmic function
    - Find the Taylor series for a trigonometric function
    - Find the Taylor series for a given function using differentiation
    - Find the radius of convergence of a Maclaurin series for a logarithmic function
    - Find the radius of convergence of a Maclaurin series for a trigonometric function
    - Find the radius of convergence of a Maclaurin series for a binomial series
- Applications of Taylor Series
  - Applications of Taylor series in integration
    - Use a Taylor series to evaluate limits
    - Use a Taylor series to evaluate nonelementary integrals
    - Use a Taylor series to estimate definite integrals within a given error

- Other applications of Taylor and power series
    - Use a Taylor series to approximate probability
    - Use a power series to solve a first order differential equation
    - Solve Airy's equation using a power series

Chapter 11: Parametric Equations and Polar Coordinates

11.1 Parametric Equations

- Graphs of Parametric Equations and Eliminating the Parameter
    - Graph parametric equations
        - Determine table values to use for drawing parametric curves by plotting points
        - Determine table values to use for drawing trigonometric parametric curves by plotting points
        - Identify a curve defined by parametric equations by plotting points
        - Identify a curve defined by trigonometric parametric equations by plotting points
        - Use technology to draw a curve defined by parametric equations
        - Identify the parametric equations of curves
    - Eliminate the parameter in parametric equations
        - Write the parametric equations of a curve in y=f(x) form by eliminating the parameter in linear equations
        - Write the parametric equations of a curve in y=f(x) form by eliminating the parameter in polynomial and radical equations
        - Write the parametric equations of a curve in y=f(x) form by eliminating the parameter in exponential and logarithmic equations
        - Write the parametric equations of a curve in y=f(x) form by eliminating the parameter in trigonometric equations
        - Determine the domain of a function whose parameter has been eliminated for linear equations
        - Determine the domain of a function whose parameter has been eliminated for trigonometric equations
    - Write parametric equations
        - Write the parametric equation of a curve
        - Write the parametric equations for a line segment given an orientation
- Cycloids and Other Parametric Curves
    - Graph parametric equations for cycloids and hypocycloids
        - Use technology to graph a cycloid
        - Use technology to graph a hypocycloid
        - Determine the number of cusps on the curve of a hypocycloid
    - Solve applications using parametric equations
        - Model motion application problems with parametric equations

- Use parametric equations to find the maximum height of an object
- Use parametric equations to find the displacement of an object

11.2 Calculus of Parametric Curves

- Derivatives of Parametric Equations
  - Find the first derivative of a parametric curve
    - Determine the slope of a line defined parametrically without eliminating the parameter
    - Determine the derivative of a parametrically defined curve with linear functions
    - Determine the derivative of a parametrically defined curve with polynomial and radical functions
    - Determine the derivative of a parametrically defined curve with exponential and logarithmic functions
    - Determine the derivative of a parametrically defined curve with trigonometric functions
  - Determine the equation of the tangent line to a parametrically defined curve
    - Determine the slope of the tangent line at a given point
    - Find the points on a parametrically defined curve that have a given slope
    - Determine the equation of the tangent to a parametrically defined curve
  - Find the second derivative of a parametric curve
    - Determine the second derivative of a parametrically defined curve with polynomial and radical functions
    - Determine the second derivative of a parametrically defined curve with exponential and logarithmic functions
    - Determine the second derivative of a parametrically defined curve with trigonometric functions
    - Determine the concavity of a parametric curve
    - Determine the critical point(s) of a parametrically defined curve
- Integrals Involving Parametric Equations
  - Find the area under a parametric curve
    - Set up a definite integral to describe the area under a parametric curve
    - Find the area under a parametric curve with linear functions
  - Use the equation for arc length for a parametric curve
    - Set up a definite integral to describe the arc length of a parametric curve
    - Use the equation for arc length for a parametric curve
  - Apply the formula for surface area to a volume generated by a parametric curve
    - Find the surface area of a known figure
    - Set up a definitely integral to find the surface area of a shape generated by revolving a curve around the x-axis

- Use technology to find the surface area of a shape generated by revolving a curve around an axis

11.3 Polar Coordinates

- Defining Polar Coordinates
  - Locate points in a plane by using polar coordinates
    - Plot a point in polar coordinates
    - Determine the polar coordinates of a point plotted on a graph
    - Determine other representations of a given polar coordinate
  - Convert points between polar and rectangular coordinates
    - Convert coordinates from polar to rectangular form
    - Convert coordinates from rectangular to polar form
- Polar Equations
  - Convert equations between polar and rectangular form
    - Convert a polar equation to rectangular form
    - Convert a rectangular equation to polar form
  - Graph polar equations
    - Find zeros and maximum values for a polar equation
    - Create a table of values for a given polar equation
    - Identify the graph of a polar equation given a table of ordered pairs
    - Identify the basic shape of the graph of a polar equation
    - Convert a rectangular equation to polar form and graph the equation using technology
    - Convert a polar equation to rectangular form and graph the equation using technology
  - Identify symmetry

11.4 Area and Arc Length in Polar Coordinates

- The Calculus of Polar Coordinates
  - Apply the formula for the area of a region in polar coordinates
    - Determine the definite integral that represents the area under a polar curve
    - Determine the area of a region bounded by a polar curve
    - Determine intersection points of two polar curves
    - Determine the area of a region bounded by two polar curves
  - Solve application problems involving the derivative of polar equations
    - Find the slope of the tangent line to a polar curve at a specified point
    - Find the points on a polar curve where the tangent line is horizontal or vertical
  - Apply the formula for the arc length of a polar curve
    - Determine the definite integral that represents the arc length of a polar curve
    - Find the arc length of a curve
    - Use technology to approximate the length of a curve over a finite interval

Knewton Alta

11.5 Conic Sections

- Conic Sections in Standard Form
  - Identify and graph a parabola given in standard form
    - Write the equation of a parabola given its directrix and focus
    - Identify the key components of a parabola
    - Graph a parabola given in standard form
    - Write the equation of a parabola in standard form
  - Identify and graph an ellipse given in standard form
    - Write the equation of an ellipse given its key components
    - Identify the key components of an ellipse
    - Graph an ellipse given in standard form
    - Write the equation of an ellipse in standard form
  - Identify and graph a hyperbola given in standard form
    - Write the equation of a hyperbola given its foci and vertices
    - Identify the key components of a hyperbola
    - Graph a hyperbola given in standard form
    - Write the equation of a hyperbola in standard form
- Eccentricity and Polar Equations of Conic Sections
  - Identify conic sections written in rectangular coordinates by eccentricity and write the equation of the conic
  - Identify conic sections written as polar equations by eccentricity and write the equation of the conic
    - Determine the eccentricity of a conic in polar coordinates
    - Recognize a conic section from its eccentricity value
    - Write the polar equation of a conic section given its eccentricity and directrix
    - Write the equation of a hyperbola given its foci and eccentricity
- General Equations of Degree Two
  - Identify key components of conic sections given as polar equations and graph them
    - Identify the vertices of a conic section written as a polar equation
    - Identify the center of a conic section written as a polar equation
    - Identify the foci of a conic section written as a polar equation
    - Identify the directrices of a conic section written as a polar equation
    - Graph a conic section given as a polar equation
  - Identify a conic given in general form
  - Identify and graph a rotated conic section
    - Calculate the angle of rotation of axes for a conic
    - Rewrite the equation of a rotated conic.
    - Graph a rotated conic
- Applications of Conic Sections

- Use applications of conic sections to solve problems
    - Use eccentricity to solve problems of planetary orbit
    - Use parabolas to solve application problems
    - Use ellipses to solve application problems

Chapter 12: Vectors in Space

12.1 Vectors in the Plane

- Introduction to Plane Vectors
    - Describe plane vectors
        - Describe a plane vector using correct notation
        - Determine if two vectors are equivalent vectors
    - Sketch a vector in two dimensions
        - Sketch a vector between points
        - Sketch a vector resulting from scalar multiplication
        - Sketch a vector resulting from vector addition
- Vectors in Component Form and Unit Vectors
    - Use vectors in component form
        - Express a vector in component form
        - Add or subtract vectors in component form
        - Perform a scalar multiplication on a vector in component form
        - Determine the magnitude of a vector
        - Find the component form of a vector using trigonometry
    - Express a vector in terms of unit vectors
        - Find a unit vector
        - Find a vector with a given magnitude and direction
        - Express a vector in terms of standard unit vectors
        - Perform operations using standard unit vectors
- Applications of Vectors
    - Solve applications using vectors
        - Find horizontal and vertical component vectors
        - Find a resultant force using vectors
        - Find the magnitude of a resultant force using vectors
        - Find the direction angle of a resultant force using vectors
        - Find a resultant velocity using vectors
        - Find the magnitude of a resultant velocity using vectors
        - Find the direction angle of a resultant velocity using vectors

12.2 Vectors in Three Dimensions

- The Three-Dimensional Coordinate System
    - Describe three dimensional space mathematically

- Identify a point in three-dimensional space
- Sketch a point in three-dimensional space
- Find the distance between two points in space
- Write equations in three dimensions
  - Write the equation of a plane parallel to a coordinate plane
  - Write the equation of a plane passing through three points in space
  - Find the equation of a sphere
  - Describe other equations in three dimensions
  - Graph other equations in three dimensions
- Vectors in Three Dimensions
  - Graph and find component forms for vectors in three dimensions
    - Sketch a vector in three-dimensional space
    - Express a vector in three dimensions in component form
    - Find an initial or terminal point on a vector
    - Express a vector in three dimensions in standard unit form
  - Perform vector operations in three dimensions
    - Perform operations on vectors in three dimensions
    - Determine the magnitude of a vector in three dimensions
    - Find a unit vector in three dimensions
    - Find vectors with a given magnitude and direction in space
- Applications of Vectors in Three Dimensions
  - Solve applications using vectors in three dimensions
    - Find a velocity vector in three dimensions
    - Find a force vector in three dimensions
    - Find the angle between a force and an axis

12.3 The Dot Product

- The Dot Product
  - Calculate the dot product
    - Calculate the dot product of two vectors in component form
    - Calculate the dot product of two vectors in unit vector form
  - Use properties of the dot product
    - Evaluate an expression containing the dot product of two vectors
    - Evaluate an expression containing the dot product of three vectors
    - Calculate the magnitude of a vector using the dot product
- Angles Between Vectors and Projections of Vectors
  - Find angles using the dot product
    - Find the measure of the angle between two vectors
    - Identify the type of angle between two vectors
    - Determine whether two given vectors are orthogonal

- Determine if two vectors are parallel, orthogonal, or neither
- Determine the real number that makes two vectors orthogonal
- Find the angle a vector makes with a unit vector
- Find the direction cosines of a vector
- Find the direction angles of a vector

- Find the projection of one vector onto another vector
  - Find the scalar projection of one vector onto another vector
  - Find the projection of one vector onto another vector
  - Write the decomposition of a vector into orthogonal components

- Applications of the Dot Product
  - Solve applications of the dot product
    - Find velocity using scalar projection
    - Calculate the work done by a constant force

12.4 The Cross Product

- The Cross Product
  - Calculate the cross product
    - Calculate the cross product of two vectors
    - Identify a vector resulting from the cross product of vectors
    - Calculate the determinant of a matrix
    - Use a determinant to calculate the cross product of two vectors
  - Use properties of the cross product
    - Find the cross product of standard unit vectors
    - Use properties of the cross product to evaluate an expression
    - Calculate the magnitude of a cross product
    - Find a unit vector in the direction of a cross product
    - Find a vector orthogonal to the plane determined by three noncollinear points
- Triple Scalar Products
  - Use properties of the triple scalar product
    - Calculate the triple scalar product
    - Determine if three vectors are coplanar
    - Determine if four points lie in the same plane
- Area and Volume with the Cross Product
  - Determine areas and volumes using the cross product
    - Find the area of a parallelogram given adjacent sides
    - Find the area of a triangle given adjacent sides
    - Find the area of a triangle given three vertices
    - Find the distance from a point to a line
    - Find the volume of a parallelepiped determined by three vectors
    - Find the volume of a parallelepiped given adjacent edges

- Applications of the Cross Product
    - Solve applications using the cross product
        - Find torque about a point
        - Find the magnitude of the torque about a point
        - Find the magnitude of the force needed to supply a given torque
        - Determine the angle between a position vector and a force given a magnitude

12.5 Equations of Lines and Planes in Space

- Equations for a Line in Space
    - Find the parametric and symmetric equations of a line
        - Find the parametric and symmetric equations of a line through a point in a given direction
        - Find the parametric and symmetric equations of a line through two points
        - Find the parametric equations of a line segment
    - Use properties of lines in space
        - Find the point of intersection between two lines
        - Determine if two lines are equal, parallel but not equal, skew, or intersecting
- Equations for a Plane
    - Write the scalar and general form of the equations of a plane
        - Write the scalar and general form of the equation of a plane through a given point with a given normal
        - Write the scalar and general form of the equation of a plane given three points
        - Write the scalar and general form of the equations of a plane given a point and a line
        - Write the scalar and general form of the equations of a plane given two lines
    - Use properties of planes in space
        - Find the distance from a point to a given plane
        - Find the line of intersection between two planes
        - Find the angle between two planes
        - Find the distance between two parallel planes

12.6 Quadric Surfaces

- Cylinders and Quadric Surfaces
    - Identify and graph cylindrical surfaces
    - Identify and graph quadric surfaces
        - Determine the axis of symmetry of a quadric surface
        - Identify the equation or traces of a quadric surface
        - Graph the traces of a quadric surface
        - Identify the graph of a quadric surface
    - Identify equations of quadric surfaces
        - Identify the surface represented by a given equation

- Write the equation of a quadric surface in standard form

12.7 Cylindrical and Spherical Coordinates

- Cylindrical and Spherical Coordinates
  - Convert between cylindrical and rectangular coordinates
    - Convert from cylindrical to rectangular coordinates when given a point
    - Convert from rectangular to cylindrical coordinates when given a point
    - Find the equation of a surface in rectangular coordinates given the equation in cylindrical coordinates
    - Identify a surface in the cylindrical coordinate system
    - Find the equation of a surface in cylindrical coordinates given the equation in rectangular coordinates
  - Convert between spherical and rectangular coordinates
    - Convert from spherical to rectangular coordinates when given a point
    - Convert from rectangular to spherical coordinates when given a point
    - Find the equation of a surface in rectangular coordinates given the equation in spherical coordinates
    - Identify a surface in the spherical coordinate system
    - Find the equation of a surface in spherical coordinates given the equation in rectangular coordinates
  - Convert between cylindrical and spherical coordinates
    - Convert from cylindrical to spherical coordinates when given a point
    - Convert from spherical to cylindrical coordinates when given an point
- Applications with Cylindrical or Spherical Coordinates
  - Solve application problems involving cylindrical and spherical coordinates
    - Convert longitude and latitude to spherical coordinates
    - Represent an object with an appropriate coordinate system

Chapter 13: Vector-Valued Functions

13.1 Vector-Valued Functions and Space Curves

- Definition and Graphs of Vector-Valued Functions
  - Write equations of vector-valued functions
    - Determine the component functions for a vector-valued function
    - Write the equation of a vector-valued function
    - Eliminate the parameter of a vector-valued function
    - Write the equation for a transformation of a vector-valued function
  - Evaluate and determine the domain of a vector-valued function
    - Evaluate a vector-valued function
    - Determine the domain of a vector-valued function
  - Graph vector-valued functions

        - Graph projections of a vector-valued curve on coordinate planes
        - Graph a vector-valued function in two dimensions
        - Graph a vector-valued function in three dimensions
- Limits and Continuity of Vector-Valued Functions
    - Determine limits and continuity of vector-valued functions
        - Calculate the limit of a vector-valued function
        - Determine continuity of a vector-valued function
- Intersections of Surfaces
    - Determine the intersection of two surfaces
        - Find a vector-valued function that represents the intersection of two surfaces
        - Sketch the space curve represented from the intersection of two surfaces

13.2 Calculus of Vector-Valued Functions

- Derivatives of Vector-Valued Functions
    - Find the derivative of vector-valued functions
        - Find the derivative of a vector-valued function
        - Find a higher-order derivative of a vector-valued function
    - Use linear properties of the derivative of vector-valued functions
        - Use the scalar multiple property of the derivative of vector-valued functions
        - Use the sum and difference properties of the derivative of vector-valued functions
        - Use the scalar product property of the derivative of vector-valued functions
    - Use dot product, cross product, and chain rule properties of derivatives of vector-valued functions
        - Use the dot product property of the derivative of vector-valued functions
        - Use the cross product property of the derivative of vector-valued functions
        - Use the chain rule property of the derivative of vector-valued functions
- Tangent Vectors and Unit Tangent Vectors
    - Compute tangent and unit tangent vectors for vector-valued functions
        - Find a tangent vector for a vector-valued function at a specified point
        - Find a unit tangent vector for a vector-valued function
        - Find a unit tangent vector for a vector-valued function at a specified point
        - Find an equation of a line tangent to a vector-valued function at a specified point
- Integrals of Vector-Valued Functions
    - Find integrals of vector-valued functions
        - Find the antiderivative of a vector-valued function given an initial condition
        - Evaluate the indefinite integral of a vector-valued function
        - Evaluate a definite integral of a vector-valued function

13.3 Arc Length and Curvature

- Arc Length and Arc-Length Parameterization

- Find the arc length for a vector-valued function
  - Write an integral to find the arc length of a curve
  - Find the arc length of a curve
- Find the arc length parametrization for a vector-valued function
  - Write an integral that represents the arc length function for a curve
  - Find the arc length function for a curve
  - Identify whether a vector-valued function is parametrized by arc length
  - Find the arc length parametrization of a curve given an arc length function
  - Find the arc length parametrization of a curve

- Curvature
  - Find the curvature of a vector-valued function
    - Find the curvature of a curve using the arc length parametrization
    - Find the curvature of a curve using a different parametrization
    - Find the curvature for a space curve using a cross-product
    - Find the curvature for a plane curve expressed as y(x)
    - Identify where the curvature of a plane curve is a minimum or maximum
- Normal and Binormal Vectors
  - Find the normal and binormal vectors for a vector-valued function
    - Find the principal unit normal vector for a curve
    - Find the binormal vector for a curve
  - Find the osculating circle and radius of curvature for a curve
    - Find the radius of curvature for a curve
    - Find the equation of the osculating circle for a plane curve at a point
    - Find the normal and osculating planes for a space curve at a point

13.4 Motion in Space

- Motion Vectors in the Plane and in Space
  - Find the velocity and acceleration vectors of particle in motion
    - Find the velocity vector of a particle moving along a curve
    - Find the speed of a particle moving along a curve
    - Identify when the speed of a moving particle is a maximum or minimum
    - Find the acceleration vector of a particle moving along a curve
    - Solve an initial value problem to find the position and velocity vectors given the acceleration vector
    - Use technology to graph the velocity and acceleration vectors for a moving particle
- Components of the Acceleration Vector
  - Find the tangential and normal components of acceleration
    - Find the tangential component of the acceleration vector of a moving particle
    - Find the normal component of the acceleration vector of a moving particle

- Decompose the acceleration vector into its tangential and normal components
- Solve problems involving centripetal force and friction

- Projectile Motion
  - Solve problems involving projectile motion
    - Find the maximum height of a projectile in motion
    - Find the horizontal range of a projectile in motion
    - Find the time at which a projectile attains a given position
    - Find the initial velocity needed for a projectile to attain a given position
    - Use Kepler's laws of planetary motion to solve problems

Chapter 14: Differentiation of Functions of Several Variables

14.1 Functions of Several Variables

- Functions of Two Variables
  - Recognize a function of two variables and identify its domain and range
    - Evaluate a function of two variables
    - Find the domain of a function of two variables
    - Find the range of a function of two variables
  - Sketch a graph of a function of two variables
    - Identify the graph of a function of two variables
    - Find the equation for the level curve of a function of two variables
    - Find the equation for the vertical trace of a function of two variables
    - Sketch the level curves of a function of two variables
    - Sketch the vertical traces of a function of two variables
- Functions of More Than Two Variables and Applications
  - Recognize a function of three or more variables and identify its level surfaces
    - Evaluate a function of three variables
    - Find the domain of a function of three variables
    - Find the equation for a level surface of a function of three variables
    - Identify the graph for a level surface of a function of three variables
  - Use functions of several variables in applications
    - Evaluate a function of two variables in applications
    - Find the equation for a level curve of a function of two variables in applications
    - Find the equation for a level surface of a function of three variables in applications

14.2 Limits and Continuity

- Limit of a Function of Two Variables
  - Use the formal definition of a limit of a function of two variables
    - Given a limit statement for a function of two variables, write the corresponding epsilon-delta statement
    - Given the epsilon-delta statement of a limit of a function of two variables, write the corresponding limit statement
  - Evaluate the limit of a function of two variables
    - Use the sum, difference, or constant multiple laws to evaluate the limit of a function of two variables

- Use the product or quotient laws to evaluate the limit of a function of two variables
- Use the power or root laws to evaluate the limit of a function of two variables
- Use algebraic techniques to evaluate the limit of a function of two variables
- Evaluate the limit of a function of two variables at a boundary point
  - Understand the difference between interior and boundary points
  - Evaluate the limit of a function of two variables at a boundary point
- Use different paths of approach to determine if the limit of a function of two variables exists
  - Evaluate the limit of a function of two variables for a particular path of approach
  - Use paths of approach to determine if the limit of a function of two variables exists

- Continuity of a Function of Two Variables
  - Understand the conditions for continuity of a function of two variables
    - Given a graph of a function of two variables, determine why the definition of continuity does not hold
    - Verify the continuity of a function of two variables at a point
    - Determine the region of the coordinate plane where a function of two variables is continuous
  - Apply the theorems of continuous functions
    - Apply the sum theorem for continuity of functions of two variables
    - Apply the product theorem for continuity of functions of two variables
    - Apply the composition theorem for continuity of functions of two variables
- Limit and Continuity of a Function of More Than Two Variables
  - Verify the continuity of the function of three variables at a point
    - Evaluate the limit of a function of three variables
    - Verify the continuity of a function of three variables at a point

14.3 Partial Derivatives

- Partial Derivatives of a Function
  - Calculate the partial derivatives of a function of two variables
    - Calculate the sign of a partial derivative using the graph of a surface
    - Calculate a first-order partial derivative of a function of two variables
    - Calculate a first-order partial derivative of a function of two variables at a point
    - Use contours to estimate a partial derivative of a function at a given point
    - Determine the slope of a surface at a point
    - Determine points in which first-order partial derivatives are equal for a given function
  - Calculate the partial derivatives of a function of more than two variables
    - Calculate a first-order partial derivative of a function of more than two variables
    - Calculate a first-order partial derivative of a function of more than two variables at a point
    - Determine the rate of change of a function with respect to a given variable
- Higher-Order Partial Derivatives and Partial Differential Equations
  - Determine the higher-order derivatives of a function of two or more variables
    - Calculate second-order partial derivatives of a function of two variables

        - Calculate higher-order partial derivatives of a function of two or more variables
    - Verify a solution to a partial differential equation
        - Verify a solution to a partial differential equation with two variables
        - Verify a solution to a partial differential equation with more than two variables

14.4 Tangent Planes and Linear Approximation

- Tangent Planes
    - Find the normal line to a surface at a point
        - Find the normal vector to a surface at a point
        - Find parametric or symmetric equations for the normal line to a surface at a point
    - Find the tangent plane to a surface at a point
        - Find the equation of the tangent plane to a surface at a point
        - Find the points on a surface where the tangent plane is horizontal
        - Find the angle of inclination of the tangent plane to a surface at a point
- Linear Approximations of Functions of Several Variables
    - Find the linear approximation of a function of two or more variables at a point
        - Find the linear approximation of a function of two variables at a point
        - Use the linear approximation of a function of two variables to estimate a function value
        - Find the linear approximation of a function of more than two variables at a point
        - Use the linear approximation of a function of more than two variables to estimate a function value
- Differentiability of a Function of Several Variables
    - Show that a function of several variables is differentiable
        - Show that a function of two variables is differentiable using the definition
        - Determine whether a function of two variables is differentiable using its partial derivatives
        - Determine whether a function of two variables is continuous
        - Determine whether a function of more than two variables is differentiable
        - Determine whether a function of more than two variables is continuous
- Differentials and Maximum Error
    - Find the differential of a function of two or more variables
        - Find the total differential of a function of two variables
        - Use a differential to estimate a change in a function of two variables
        - Find the total differential of a function of more than two variables
        - Use a differential to estimate a change in a function of more than two variables
    - Use a differential to estimate a change in a geometric application
        - Use a differential to estimate a change in area or surface area
        - Use a differential to estimate a change in volume
    - Use a differential to estimate the error introduced by an error in measurement
        - Use a differential to estimate the propagated error
        - Use a differential to estimate the relative or percent error

14.5 The Chain Rule

- The Chain Rule for One or Two Independent Variables
    - Use the chain rule for one independent variable

        - Calculate the derivative of a function with one independent variable using the chain rule
        - Find the rate of change of a function with one independent variable involving a real-world application
    - Use the chain rule for several independent variables
        - Calculate a partial derivative of a function with two independent variables using the chain rule
        - Calculate a partial derivative of a function with several independent variables using the generalized chain rule
- Implicit Differentiation by Partial Derivatives
    - Use implicit differentiation to calculate partial derivatives
        - Use implicit differentiation to calculate the derivative of an implicitly defined function of two variables
        - Use implicit differentiation to calculate the partial derivative of an implicitly defined function of three variables

14.6 Directional Derivatives and the Gradient

- Directional Derivatives and Gradients for Functions in Two Variables
    - Find a directional derivative for a function of two variables
        - Find a directional derivative of a function of two variables using the limit definition
        - Find a directional derivative of a function of two variables in the direction of a given angle
        - Find a directional derivative of a function of two variables in the direction of a given vector
        - Find a directional derivative of a function of two variables at a point in the direction of another point
    - Find and use the gradient of a function of two variables
        - Find the gradient of a function of two variables
        - Use the gradient to find a directional derivative for a function of two variables
- Applications of Directional Derivatives and Gradients
    - Use the gradient to maximize or minimize a directional derivative for a function of two variables
        - Find the maximum or minimum value of a directional derivative of a function of two variables
        - Find the direction in which a directional derivative is maximized or minimized for a function of two variables
    - Find normal and tangent vectors to a level curve of a function of two variables
        - Find a normal vector to a level curve of a function of two variables
        - Find a tangent vector to a level curve of a function of two variables
- Directional Derivatives and Gradients for Functions in Three Variables
    - Find and use the gradient of a function of three variables
        - Find the gradient of a function of three variables
        - Find a directional derivative of a function of three variables
    - Use the gradient to maximize or minimize a directional derivative for a function of three variables

        - Find the maximum or minimum value of a directional derivative of a function of three variables
        - Find the direction in which a directional derivative is maximized or minimized for a function of three variables
- Applications of Functions of Two or More Variables
    - Solve application problems for functions of two or more variables
        - Solve problems involving temperature for a function of two variables
        - Solve problems involving temperature for a function of three variables
        - Solve problems involving topographical maps for a function of two variables
        - Solve problems involving topographical maps for a function of three variables
        - Solve problems involving electrical potential for a function of two or more variables
        - Solve problems involving electrical potential for a function of three variables

14.7 Maxima/Minima Problems

- Critical Points and the Second Derivative Test for Functions of Several Variables
    - Use partial derivatives to locate critical points of a function of two variables
        - Find the critical points of a polynomial function using partial derivatives
        - Find the critical points of a radical function using partial derivatives
        - Find the critical points of an exponential function using partial derivatives
    - Use the second derivative test to classify critical points of a function of two variables
        - Given a critical point and a function, classify the point as a local maximum, a local minimum, or a saddle point
        - Use the second derivative test to find the local extrema of a given function
        - Find the absolute extrema of a function over a closed, rectangular region
        - Find the absolute extrema of a function over a closed disk
- Maximization and Minimization Problems for Functions of Several Variables
    - Solve maximization and minimization problems that involve functions of several variables
        - Find three positive numbers that satisfy given conditions
        - Find points on a plane or surface closest to a given point
        - Determine the maximum or minimum volume of a given object
        - Solve maximum revenue and profit problems

14.8 Lagrange Multipliers

- Lagrange Multipliers
    - Use the method of Lagrange multipliers to optimize a function of two variables
        - Graphically identify maximum and minimum values of a function of two variables given a constraint
        - Use the method of Lagrange multipliers to optimize a function of two variables with one constraint
        - Determine the absolute maximum and minimum values of a function using Lagrange multipliers on the boundary
    - Use the method of Lagrange multiples to optimize a function of three variables
        - Use the method of Lagrange multiples to optimize a function of three variables with one constraint
        - Use the method of Lagrange multiples to optimize a function of three variables with two constraints

- Applications Using Lagrange Multipliers
  - Use the method of Lagrange multipliers to solve optimization problems with geometric applications
    - Use the method of Lagrange multipliers to optimize the distance between geometric objects
    - Use the method of Lagrange multipliers to optimize a geometric feature of an inscribed figure
    - Use the method of Lagrange multipliers to optimize one geometric feature given a constraint on another
    - Use the method of Lagrange multipliers to find the highest point on the intersection of two surfaces
  - Use the method of Lagrange multipliers to solve optimization problems with business applications
    - Use the method of Lagrange multipliers to minimize a cost function
    - Use the method of Lagrange multipliers to maximize a profit function
    - Use the method of Lagrange multipliers to optimize production level

Chapter 15: Multiple Integration

15.1 Double Integrals over Rectangular Regions

- Double Riemann Sums
  - Use a double Riemann sum to approximate a double integral over a rectangular region
    - Use a double Riemann sum to approximate a double integral given a table of values for a function
    - Use a double Riemann sum to approximate a double integral given the level curves for a function
    - Use a double Riemann sum to approximate a double integral of a polynomial function
    - Use a double Riemann sum to approximate a double integral of a trigonometric function
- Fubini's Theorem
  - Use Fubini's theorem to set up an iterated double integral over a rectangular region
    - Set up an iterated double integral of a polynomial function over a rectangular region
    - Set up an iterated double integral of a radical function over a rectangular region
    - Set up an iterated double integral of a trigonometric function over a rectangular region
    - Set up an iterated double integral of an exponential function over a rectangular region
    - Set up an iterated double integral of a logarithmic function over a rectangular region
    - Set up an iterated double integral of an inverse trigonometric function over a rectangular region
  - Use Fubini's theorem to evaluate an iterated double integral over a rectangular region
    - Evaluate an iterated double integral of a polynomial function over a rectangular region

        - Evaluate an iterated double integral of a radical function over a rectangular region
        - Evaluate an iterated double integral of a trigonometric function over a rectangular region
        - Evaluate an iterated double integral of a exponential function over a rectangular region
        - Evaluate an iterated double integral of a logarithmic function over a rectangular region
        - Evaluate an iterated double integral of an inverse trigonometric function over a rectangular region
- Switching the Order of Integration
    - Change the order of integration of an iterated double integral over a rectangular region
        - Change the order of integration of an iterated double integral over a rectangular region
        - Change the order of integration to evaluate a double integral of a polynomial function
        - Change the order of integration to evaluate a double integral of a radical function
        - Change the order of integration to evaluate a double integral of an exponential function
- Properties of Double Integrals
    - Understand the properties of double integrals
        - Evaluate a linear combination of double integrals over a rectangular region
        - Evaluate a double integral over a union of adjacent rectangular regions
        - Evaluate a double integral of a polynomial function that can be written as a product of two integrals
        - Evaluate a double integral of a radical function that can be written as a product of two integrals
        - Evaluate a double integral of a trigonometric function that can be written as a product of two integrals
        - Evaluate a double integral of an exponential function that can be written as a product of two integrals
        - Evaluate a double integral of a logarithmic function that can be written as a product of two integrals
    - Find upper and lower bounds on the value of a double integral
        - Find upper and lower bounds on the value of a double integral of a polynomial function
        - Find upper and lower bounds on the value of a double integral of a trigonometric function
        - Find upper and lower bounds on the value of a double integral of an exponential function
- Applications of Double Integrals Over Rectangular Regions
    - Use a double Riemann sum to approximate the volume of a solid with a rectangular base
        - Use a double Riemann sum to approximate the volume of a solid with a rectangular base given a table of values

        - Use a double Riemann sum to approximate the volume of a solid with a rectangular base bounded above by a polynomial function
        - Use a double Riemann sum to approximate the volume of a solid with a rectangular base bounded above by a trigonometric function
    - Use a double integral to determine the volume of a solid with a rectangular base
        - Use a double integral to determine the volume of a solid with a rectangular base bounded above by a plane
        - Use a double integral to determine the volume of a solid with a rectangular base bounded above by an elliptic paraboloid
        - Use a double integral to determine the volume of a solid with a rectangular base bounded above by a trigonometric function
        - Use a double integral to determine the volume of a solid with a rectangular base bounded above by an exponential function
- Average Value of a Function of Two Variables
    - Use a double Riemann sum to approximate the average value of a function of two variables over a rectangular region
        - Use a double Riemann sum to approximate the average value of a function over a rectangular region given a table of values
    - Use a double integral to determine the average value of a function of two variables over a rectangular region
        - Use a double integral to determine the average value of a polynomial function over a rectangular region
        - Use a double integral to determine the average value of a trigonometric function over a rectangular region
        - Use a double integral to determine the average value of an exponential function over a rectangular region
        - Use a double integral to determine the average value of an inverse trigonometric function over a rectangular region
        - Use a double integral to determine the average squared distance from a point to a rectangular region

15.2 Double Integrals over General Regions

- General Regions of Integration
    - Recognize when a function of two variables is integrable over a general region
        - Determine whether a region bounded by linear functions is of Type I or Type II
        - Determine whether a region bounded by two quadratic functions is of Type I or Type II
        - Determine whether a region bounded by two trigonometric functions is of Type I or Type II
        - Determine whether a region bounded by a trigonometric and a polynomial function is of Type I or Type II
        - Determine whether a region bounded by a quadratic and a radical function is of Type I or Type II
    - Set up an iterated double integral for a function of two variables over a general region
        - Set up an iterated double integral of a polynomial function over a region of Type I

        - Set up an iterated double integral of an exponential or logarithmic function over a region of Type I
        - Set up an iterated double integral of a polynomial function over a region of Type II
        - Set up an iterated double integral of an exponential or logarithmic function over a region of Type II
- Double Integrals Over Non-Rectangular Regions
    - Evaluate a double integral using Fubini's theorem
        - Evaluate a double integral over a polynomial function region of Type I
        - Evaluate a double integral over a trigonometric function region of Type I
        - Evaluate a double integral over an inverse trigonometric function region of Type II
        - Evaluate a double integral over a polynomial and radical function region of Type II
        - Evaluate a double integral over a polynomial function region of Type II
- Double Integrals by Decomposing Regions
    - Evaluate a double integral over a complex region by decomposing the region
        - Evaluate a double integral of a polynomial function over a more complex exponential function and polynomial function region using a union of regions of Type I
        - Evaluate a double integral of a polynomial function over a more complex polynomial function region using a union of regions of Type I
        - Evaluate a double integral of a polynomial function over a more complex polynomial function region using a union of regions of Type II
        - Evaluate a double integral of a polynomial function over a complex triangular region using a union of regions of Type I
        - Evaluate a double integral of a polynomial function over a complex quadrilateral region using a union of regions of Type I
        - Evaluate a double integral of a polynomial function over a complex parabolic region using a union of regions of Type I
        - Evaluate a double integral of an exponential function over an algebraic region using a union of regions of Type I
        - Evaluate a double integral a polynomial function over a more complex polynomial function region using a union of regions of Type II
- Double Integrals by Changing the Order of Integration
    - Simplify the calculation of an iterated integral by changing the order of integration
        - Reverse the order of integration for an iterated double integral over a polynomial function region
        - Reverse the order of integration for an iterated double integral over a triangular region
        - Evaluate a double integral of a polynomial function over a polynomial region by using the easier order of integration
        - Evaluate a double integral of a polynomial function over a parabolic region by using the easier order of integration

        - Evaluate a double integral of a rational function over an exponential function and polynomial function region by using the easier order of integration
        - Evaluate a double integral of a polynomial function over a trigonometric region by using the easier order of integration
        - Evaluate a double integral of a trigonometric function by changing the region from Type I to Type II
        - Evaluate a double integral of a polynomial function over a region defined by radical functions by changing the region from Type II to Type I
        - Evaluate a double integral of a trigonometric function by changing the region from Type II to Type I
- Calculating Volumes with Double Integrals
    - Set up an iterated double integral to calculate the volume of a region between two surfaces
        - Set up an iterated double integral to find the volume of a solid in the first octant under a plane
        - Set up an iterated double integral to find the volume bounded by two cylinders and two planes
    - Use double integrals to calculate the volume of a region between two surfaces
        - Use a double integral to find the volume of a solid in the first octant under a plane
        - Use a double integral to find the volume of a solid between a planar region and a sphere
- Calculating Areas with Double Integrals
    - Set up an iterated double integral to calculate the area of a general plane region
        - Set up an iterated double integral to find the area of a region bounded by two polynomial functions
    - Use double integrals to calculate the area of a general plane region
        - Use a double integral to find the area of a region bounded by two polynomial functions by decomposing the region
        - Use a double integral to find the area of a region bounded by two trigonometric functions
        - Use a double integral to find the area of a region bounded by two logarithmic functions
- Calculating Average Values with Double Integrals
    - Find the average value of a function over a general region
        - Set up an iterated double integral to find the average value of a linear function on a triangular region
        - Set up an iterated double integral of a polynomial function to find the average value of a function over a region bounded by a parabola and a line
        - Use a double integral to find the average value of a polynomial function over a region bounded by two radical functions
- Improper Double Integrals
    - Evaluate a double improper integral
        - Use Fubini's theorem to evaluate an improper double integral of an exponential function over a polynomial region

- Use Fubini's theorem to evaluate an improper double integral of a radical function over a circular region
- Use Fubini's theorem to evaluate an improper double integral of a rational function over a radical region
- Evaluate an improper integral of a rational function over an unbounded region
- Evaluate an improper double integral of a rational function over an unbounded region
- Evaluate an improper double integral of an exponential function over an unbounded region

- Applications of Double Integrals in Probability
  - Find the probability and the expected value given a joint density function
    - Use a double integral to find a probability given an exponential joint density function
    - Use a double integral to find a probability given a constant joint density function
    - Use a double integral to find the expected value of Y given a polynomial joint density function
    - Use a double integral to find the expected value of X given a polynomial joint density function
    - Use a double integral to find the expected time given an exponential joint density function

15.3 Double Integrals in Polar Coordinates

- Double Integrals over Polar Rectangular Regions
  - Express a region of the plane in terms of polar coordinates
    - Express a region represented by a general description in polar coordinates
    - Express a region represented by boundary equations or inequalities in polar coordinates
    - Express a region represented by a graph in polar coordinates
  - Evaluate a double integral over a polar rectangular region
    - Set up an iterated integral of an algebraic function over a polar rectangular region
    - Set up an iterated integral of a transcendental function over a polar rectangular region
    - Evaluate an iterated integral of an algebraic function over a polar rectangular region
    - Evaluate an iterated integral of a transcendental function over a polar rectangular region
- Converting Double Integrals in Rectangular Coordinates to Polar Coordinates
  - Convert a double integral in rectangular coordinates to polar coordinates
    - Set up an iterated integral of a polynomial function by converting to polar coordinates
    - Evaluate an iterated integral by first converting to polar coordinates
    - Convert an integral of a polynomial function from rectangular to polar coordinates over a polar rectangle
    - Convert an integral of a trigonometric function from rectangular to polar coordinates over a polar rectangle

- Convert an integral of a polynomial function from rectangular to polar coordinates over a polar rectangle and evaluate
- Convert an integral of a trigonometric function from rectangular to polar coordinates over a polar rectangle and evaluate

- Double Integrals Over General Polar Regions
  - Evaluate a double integral over a general polar region
    - Set up a double integral in polar coordinates over a general polar region
    - Evaluate a double integral in polar coordinates over a general polar region
    - Convert an integral of a polynomial function or radical function from rectangular to polar coordinates over a general polar region
    - Convert an integral of a rational function from rectangular to polar coordinates over a general polar region
    - Convert an integral of a rational function from rectangular to polar coordinates over a general polar region and evaluate
- Polar Areas and Volumes
  - Use double integrals in polar coordinates to calculate areas
    - Use double integrals in polar coordinates to compute the area of a region bounded by a polar curve
    - Use double integrals in polar coordinates to compute the area of a region bounded between polar curves
  - Use double integrals in polar coordinates to calculate volumes
    - Set up a double integral in polar coordinates to compute the volume of a region bounded by a surface and a plane
    - Use double integrals in polar coordinates to compute the volume of a region bounded by a surface and a plane
    - Use double integrals in polar coordinates to compute the volume of a region bounded by two surfaces
- Improper Double Integrals in Polar Coordinates
  - Evaluate a double improper integral in polar coordinates
    - Use double integrals in polar coordinates to compute an improper integral of an exponential function

15.4 Triple Integrals

- Triple Integrals Over a Rectangular Box
  - Evaluate a triple integral over a rectangular box
    - Evaluate a triple iterated integral of a polynomial function over a rectangular box
    - Evaluate a triple iterated integral of a rational function over a rectangular box
    - Evaluate a triple iterated integral of a trigonometric function over a rectangular box
    - Evaluate a triple iterated integral of an exponential function over a rectangular box
    - Evaluate a triple iterated integral of a logarithmic function over a rectangular box
- Triple Integrals Over a General Bounded Region
  - Evaluate a triple integral over a general region that has a rectangular projection
    - Evaluate a triple integral of an algebraic function of Type 1 over a general region that has a rectangular projection

        - Evaluate a triple integral of Type 2 over a general region that has a rectangular projection
        - Evaluate a triple integral of Type 3 over a general region that has a rectangular projection
    - Evaluate a triple integral over a general region that has a non-rectangular projection
        - Evaluate a triple integral of Type 1 over a general region that has a non-rectangular projection
        - Evaluate a triple integral of Type 2 over a general region that has a non-rectangular projection
        - Evaluate a triple integral of Type 3 over a general region that has a non-rectangular projection
    - Evaluate a triple integral over a general region given a description of the region
        - Set a triple integral over a general region given a description of the region
        - Evaluate a triple integral over a general region given a description of the region
- Changing the Order of Integration of Triple Integral
    - Rewrite a triple integral over a rectangular box by changing the order of integration
        - Rewrite a triple integral of an algebraic function over a rectangular box by changing the order of integration
        - Rewrite a triple integral of a transcendental function over a rectangular box by changing the order of integration
    - Evaluate a triple integral over a rectangular box by changing the order of integration
        - Evaluate a triple integral of a polynomial function over a rectangular box by changing the order of integration
        - Evaluate a triple integral of a rational function over a rectangular box by changing the order of integration
        - Evaluate a triple integral of a trigonometric function over a rectangular box by changing the order of integration
        - Evaluate a triple integral of an exponential function over a rectangular box by changing the order of integration
- Volume and Average Value of a Function of Three Variables
    - Use a triple integral to find a volume of a solid bounded region
        - Set up a triple integral to find the volume of a solid
        - Evaluate a triple integral to find the volume of a solid
    - Use a triple integral to find the average value of a function of three variables
        - Use a triple integral to find the average value of an algebraic function of three variables
        - Use a triple integral to find the average value of a transcendental function of three variables

15.5 Triple Integrals in Cylindrical and Spherical Coordinates

- Integration in Cylindrical Coordinates
    - Evaluate a triple integral in cylindrical coordinates
        - Evaluate a triple integral over a cylindrical box
        - Set up a triple integral in cylindrical coordinates over a general region
    - Convert a triple integral in rectangular coordinates to cylindrical coordinates
        - Convert a triple integral in rectangular coordinates to cylindrical coordinates

- Convert a triple integral in rectangular coordinates to cylindrical coordinates and evaluate
- Integration in Spherical Coordinates
  - Evaluate a triple integral in spherical coordinates
    - Evaluate a triple integral over a spherical box
    - Set up a triple integral over a spherical box
  - Convert a triple integral in rectangular coordinates to spherical coordinates
    - Convert a triple integral of a polynomial function in rectangular coordinates to spherical coordinates
    - Convert a triple integral of a radical function in rectangular coordinates to spherical coordinates
    - Convert a triple integral of a polynomial function in rectangular coordinates to spherical coordinates and evaluate
    - Convert a triple integral of a radical function in rectangular coordinates to spherical coordinates and evaluate
- Volume in Cylindrical Coordinates
  - Find the volume of a solid using a triple integral in cylindrical coordinates
    - Set up a triple integral in cylindrical coordinates to find the volume of a solid whose boundaries are given in rectangular coordinates
    - Find the volume of a solid using a triple integral in cylindrical coordinates whose boundaries are given in rectangular coordinates
- Volume in Spherical Coordinates
  - Find the volume of a solid using a triple integral in spherical coordinates
    - Find the volume of a solid using a triple integral in spherical coordinates
    - Find the volume of a solid using a triple integral in spherical coordinates whose boundaries are given in spherical coordinates
    - Set up a triple integral in spherical coordinates to find the volume of a solid whose boundaries are given in rectangular coordinates
    - Find the volume of a solid using a triple integral in spherical coordinates whose boundaries are given in rectangular coordinates
  - Find the volume of a solid between two surfaces using a triple integral in spherical coordinates
    - Set up a triple integral in spherical coordinates to find the volume between two surfaces given in rectangular coordinates
    - Find the volume between two surfaces given in rectangular coordinates using a triple integral in spherical coordinates

15.6 Calculating Centers of Mass and Moments of Inertia

- Mass in Two Dimensions
  - Use double integrals to find the mass of a lamina
    - Use double integrals to find the mass of a rectangular lamina
    - Use double integrals to find the mass of a triangular lamina
    - Use double integrals to find the mass of a trapezoidal lamina
    - Use double integrals to find the mass of a circular lamina
    - Use double integrals to find the mass of an elliptical lamina
- Center of Mass in Two Dimensions

- Use double integrals to find the moments of a lamina
  - Use double integrals to find the moments of a rectangular lamina
  - Use double integrals to find the moments of a triangular lamina
  - Use double integrals to find the moments of a trapezoidal lamina
  - Use double integrals and technology to find the moments of a circular lamina
  - Use double integrals to find the moments of an elliptical lamina
- Use double integrals to find the center of mass of a lamina
  - Use double integrals to find the center of mass of a rectangular lamina
  - Use double integrals to find the center of mass of a triangular lamina
  - Use double integrals to find the center of mass of a trapezoidal lamina
  - Use double integrals and technology to find the center of mass of a circular lamina
  - Use double integrals to find the center of mass of an elliptical lamina
- Use double integrals to find the centroid of a lamina
  - Use double integrals to find the centroid of a triangular lamina
  - Use double integrals to find the centroid of a trapezoidal lamina
  - Use double integrals to find the centroid of an elliptical lamina

- Moments of Inertia in Two Dimensions
  - Use double integrals to find the moments of inertia of a lamina
    - Use double integrals to find the moments of inertia of a rectangular lamina
    - Use double integrals to find the moments of inertia of a triangular lamina
    - Use double integrals to find the moments of inertia of a trapezoidal lamina
    - Use double integrals and technology to find the moments of inertia of a circular lamina
    - Use double integrals and technology to find the moments of inertia of an elliptical lamina
  - Use double integrals to find the radius of gyration of a lamina
    - Use double integrals to find the radius of gyration of a rectangular lamina
    - Use double integrals to find the radius of gyration of a triangular lamina
    - Use double integrals to find the radius of gyration of a trapezoidal lamina
    - Use double integrals and technology to find the radius of gyration of a circular lamina
    - Use double integrals and technology to find the radius of gyration of an elliptical lamina
- Center of Mass in Three Dimensions
  - Use triple integrals to find the mass and center of mass of a solid
    - Use triple integrals to find the mass of a solid
    - Use triple integrals to find the moments of a solid
    - Use triple integrals to find the center of mass of a solid
    - Use triple integrals to find the centroid of a solid
- Moments of Inertia in Three Dimensions
  - Use triple integrals to find the moments of inertia of a solid
    - Use triple integrals to find the moments of inertia of a solid

15.7 Change of Variables in Multiple Integrals

- Transformations of Variables

- Understand the concept of a transformation of variables
  - Determine the image of a point under a given transformation of variables in two dimensions
  - Determine the image of a region under a given transformation of variables in two dimensions
  - Determine the image of a region under a given transformation of variables in three dimensions
  - Sketch the image of a region under a given transformation of variables in two dimensions
  - Determine a transformation that maps one region to another region in two dimensions
- One-to-one and Inverse Transformations
  - Understand one-to-one and inverse transformations
    - Determine if a transformation in two dimensions is one-to-one
    - Determine if a transformation in three dimensions is one-to-one
    - Determine an inverse transformation for a given one-to-one transformation containing a linear function in two dimensions
    - Determine an inverse transformation for a given one-to-one transformation containing a linear function in three dimensions
    - Determine an inverse transformation for a given one-to-one transformation containing a transcendental function in two dimensions
    - Determine the preimage of a region under a given one-to-one transformation in two dimensions
    - Determine the preimage of a region under a given one-to-one transformation in three dimensions
    - Determine the image of a region using the inverse of a given one-to-one transformation
    - Sketch the image of a region using the inverse of a given one-to-one transformation of variables in two dimensions
    - Use an inverse transformation to determine a transformation that maps one region to another region in two dimensions
- Jacobians for Functions of Two Variables
  - Find the Jacobian for a transformation of a function of two variables
    - Find the Jacobian for a transformation of an algebraic function of two variables
    - Find the Jacobian for a transformation of an exponential function of two variables
    - Find the Jacobian for a transformation of a trigonometric function of two variables
- Change of Variables for Double Integrals
  - Set up a double integral of a function using a change of variables
    - Set up a double integral of a polynomial function using a change of variables with a given transformation
    - Set up a double integral of a radical function using a change of variables with a given transformation

        - Set up a double integral of a trigonometric function using a change of variables with a given transformation
        - Set up a double integral of an exponential function using a change of variables with a given transformation
    - Evaluate a double integral using a change of variables
        - Evaluate a double integral of a polynomial function using a change of variables with a given transformation
        - Evaluate a double integral of a radical function using a change of variables with a given transformation
        - Evaluate a double integral of a trigonometric function using a change of variables with a given transformation
        - Evaluate a double integral of an exponential function using a change of variables with a given transformation
        - Evaluate a double integral of a function using an appropriate change of variables
- Jacobians for Functions of Three Variables
    - Find the Jacobian for a transformation of a function of three variables
        - Find the Jacobian for a transformation of an algebraic function of three variables
        - Find the Jacobian for a transformation of a trigonometric function of three variables
- Change of Variables for Triple Integrals
    - Set up a triple integral of a function using a change of variables
        - Setup a triple integral of a polynomial function using a change of variables with a given transformation
        - Setup a triple integral of a radical function using a change of variables with a given transformation
        - Setup a triple integral of an exponential function using a change of variables with a given transformation
    - Evaluate a triple integral using a change of variables
        - Evaluate a triple integral of a polynomial function using a change of variables with a given transformation
        - Evaluate a triple integral of a radical function using a change of variables with a given transformation
        - Evaluate a triple integral of an exponential function using a change of variables with a given transformation
- Finding Area and Volume Using a Change of Variables
    - Find area using a double integral with a change of variables
        - Set up a double integral using a change of variables to find the area of a region
        - Evaluate a double integral using a change of variables to find the area of a region
    - Find volume using a change of variables
        - Evaluate a double integral using a change of variables to find the volume of a solid
        - Evaluate a triple integral using a change of variables to find the volume of a solid

Knewton Alta

Chapter 16: Vector Calculus

16.1 Vector Fields

- Properties of Vector Fields
  - Find the vector associated with a given point in a vector field in two dimensions
    - Find the vector associated with a given point in a vector field defined by polynomial functions of two variables
    - Find the vector associated with a given point in a vector field defined by rational functions of two variables
    - Find the vector associated with a given point in a vector field defined by radical functions of two variables
    - Find the vector associated with a given point in a vector field defined by trigonometric functions of two variables
    - Find the vector associated with a given point in a vector field defined by exponential functions of two variables
  - Find the vector associated with a given point in a vector field in three dimensions
    - Find the vector associated with a given point in a vector field defined by polynomial functions of three variables
    - Find the vector associated with a given point in a vector field defined by rational functions of three variables
    - Find the vector associated with a given point in a vector field defined by radical functions of three variables
    - Find the vector associated with a given point in a vector field defined by trigonometric functions of three variables
    - Find the vector associated with a given point in a vector field defined by exponential functions of three variables
  - Use properties of vector fields
    - Determine whether a vector field is radial, rotational, or neither
    - Determine whether a vector field is a unit vector field
- Draw A Vector Field
  - Sketch a vector field from a given equation in two dimensions
    - Identify the plot of a two-dimensional vector field
    - Sketch a two-dimensional radial vector field
    - Sketch a two-dimensional rotational vector field
    - Sketch a two-dimensional vector field defined by polynomial functions
    - Sketch a two-dimensional vector field defined by rational functions
    - Sketch a two-dimensional vector field defined by trigonometric functions
  - Sketch a vector field from a given equation in three dimensions
    - Identify the plot of a three-dimensional vector field
    - Sketch a three-dimensional vector field defined by polynomial functions
- Gradient Fields
  - Find the gradient vector field for a given function of two variables
    - Find the gradient vector field of an algebraic function of two variables
    - Find the gradient vector field of a transcendental function of two variables
  - Sketch the gradient vector field for a given function in two dimensions
    - Identify the plot of a gradient vector field in two dimensions

        - Sketch the plot of a gradient vector field of an algebraic function in two dimensions
        - Sketch the plot of a gradient vector field of a transcendental function in two dimensions
    - Find the gradient vector field for a given function of three variables
        - Find the gradient vector field of an algebraic function of three variables
        - Find the gradient vector field of a transcendental function of three variables
    - Determine if a vector field is not conservative
        - Use the cross-partial property to determine if a vector field defined by algebraic functions in two dimensions is not conservative
        - Use the cross-partial property to determine if a vector field defined by transcendental functions in two dimensions is not conservative
        - Use the cross-partial property to determine if a vector field defined by algebraic functions in three dimensions is not conservative
        - Use the cross-partial property to determine if a vector field defined by transcendental functions in three dimensions is not conservative

16.2 Line Integrals

- Scalar Line Integrals
    - Convert a line integral to an ordinary integral
        - Find a parameterization of a line segment
        - Find a parameterization of a quarter-circle
        - Convert a line integral of a polynomial function to an ordinary integral
        - Convert a line integral over a line segment to an ordinary integral
    - Evaluate a scalar line integral
        - Evaluate a scalar line integral of a polynomial function over a parameterized line segment
        - Evaluate scalar line integral of a trigonometric function over a parameterized trigonometric curve
        - Evaluate a scalar line integral over a parameterized trigonometric curve in three dimensions
        - Evaluate a scalar line integral of a polynomial function over a parabola
        - Evaluate a scalar line integral of a polynomial function over a half circle
    - Use a line integral to find an arclength
        - Calculate arc length of a parameterized trigonometric curve
        - Calculate arc length of a parameterized polynomial curve in two and three dimensions
- Vector Line Integrals
    - Calculate a vector line integral
        - Calculate a vector line integral of a polynomial vector field along a parameterized polynomial curve
        - Calculate a vector line integral of a rational function along a parameterized polynomial curve
        - Set up a vector line integral written in expanded form over a parameterized trigonometric curve in three dimensions.

        - Calculate a vector line integral written in expanded form over a parameterized polynomial curve
    - Use properties to compute a vector line integral
        - Use properties to compute a vector line integral of a vector field over a rectangle
        - Use properties to compute an equivalent vector line integral
- Flux and Circulation
    - Compute a Flux of a vector field
        - Compute the flux of a polynomial vector field across a line segment
        - Compute the flux of a vector field across a parameterized curve
        - Compute the flux of a vector field across a circle
    - Calculate a Circulation of a vector field
        - Calculate the circulation of a polynomial vector field F moving counterclockwise along a circle
        - Calculate the circulation of a rational vector field F along a circle
- Applications of Line Integrals
    - Calculate mass of a wire
        - Calculate the mass of a wire that follows a circle in two dimensions
        - Calculate the mass of a wire that follows a parameterized polynomial curve in three dimensions
        - Calculate the mass of a wire that follows a parameterized trigonometric curve in three dimensions
    - Find work done by a force field
        - Find the work done by a force field F on a particle moving along a three-dimensional path
        - Find the work done by a force field F on a particle moving along a line segment in three dimensions
        - Use properties of vector line integrals to describe work done along a path

16.3 Conservative Vector Fields

- Curves and Regions
    - Describe simple and closed curves
        - Identify simple and closed curves involving polynomial parameterizations
        - Identify simple and closed curves involving trigonometric parameterizations
        - Identify simple and closed curves involving a combination of polynomial and trigonometric parameterizations
    - Define connected and simply connected regions
        - Identify simply connected regions involving trigonometric curves
        - Identify simply connected regions involving polar curves
        - Identify simply connected regions involving circles
- The Fundamental Theorem for Line Integrals
    - Use the Fundamental Theorem for Line Integrals to evaluate a line integral in a vector field given a potential function
        - Evaluate a line integral in two dimensions using the Fundamental Theorem of Line Integrals involving an algebraic vector field
        - Evaluate a line integral in three dimensions using the Fundamental Theorem of Line Integrals involving an algebraic vector field

        - Evaluate a line integral in two dimensions using the Fundamental Theorem of Line Integrals involving a transcendental vector field
        - Evaluate a line integral in three dimensions using the Fundamental Theorem of Line Integrals involving a transcendental vector field
    - Determine if a vector field is conservative on a given region
        - Determine if a vector field in two dimensions involving algebraic functions is conservative on a given region
        - Determine if a vector field in three dimensions involving algebraic functions is conservative on a given region
        - Determine if a vector field in two dimensions involving transcendental functions is conservative on a given region
        - Determine if a vector field in three dimensions involving transcendental functions is conservative on a given region
    - Find a potential function for a simply connected vector field
        - Find a potential function for an algebraic vector field in two dimensions
        - Find a potential function for an algebraic vector field in three dimensions
        - Find a potential function for a transcendental vector field in two dimensions
        - Find a potential function for a transcendental vector field in three dimensions
    - Use the Fundamental Theorem of Line Integrals to evaluate a line integral
        - Given an algebraic vector field in two dimensions, find the potential function and use it to compute a line integral
        - Given an algebraic vector field in three dimensions, find the potential function and use it to compute a line integral
        - Given a transcendental vector field in two dimensions, find the potential function and use it to compute a line integral
        - Given a transcendental vector field in three dimensions, find the potential function and use it to compute a line integral

16.4 Green’s Theorem

- The Circulation Form of Green's Theorem
    - Use the circulation form of Green’s Theorem to evaluate line integrals
        - Verify the conditions of Green's Theorem
        - Use Green's Theorem to evaluate a line integral along a closed rectangular path through a vector field involving algebraic functions
        - Use Green's Theorem to evaluate a line integral along a closed rectangular path through a vector field involving transcendental functions
        - Use Green's Theorem to evaluate a line integral along a closed circular path through a vector field involving algebraic functions
        - Use Green's Theorem to evaluate a line integral along a closed circular path through a vector field involving transcendental functions
        - Use Green's Theorem to evaluate a line integral along a closed triangular path through a vector field involving algebraic functions
        - Use Green's Theorem to evaluate a line integral along a closed triangular path through a vector field involving transcendental functions
        - Use Green's Theorem to evaluate a line integral along a closed elliptical path
        - Use Green's Theorem to evaluate a line integral along a general closed path

Knewton Alta

- The Flux Form of Green's Theorem
  - Use the flux form of Green's Theorem
    - Use Green's Theorem to find the flux across a rectangle through a vector field involving algebraic functions
    - Use Green's Theorem to find the flux across a rectangle through a vector field involving transcendental functions
    - Use Green's Theorem to find the flux across a circle through a vector field involving algebraic functions
    - Use Green's Theorem to find the flux across a circle through a vector field involving transcendental functions
    - Use Green's Theorem to find the flux across a triangle through a vector field involving algebraic functions
    - Use Green's Theorem to find the flux across a triangle through a vector field involving transcendental functions
    - Use Green's Theorem to find the flux across an ellipse
- Source Free Vector Fields
  - Understand source free vector fields
    - Determine if a vector field is source free
    - Find a stream function for a source free vector field involving polynomial functions
    - Find a stream function for a source free vector field involving transcendental functions
    - Determine if a polynomial function satisfies Laplace's equation
    - Determine if a transcendental function satisfies Laplace's equation
    - Find a potential function that satisfies Laplace's equation for a source free and conservative vector field
- Green's Theorem on General Regions
  - Use Green's Theorem on general regions
    - Use Green's Theorem to evaluate a line integral on a region with holes through a vector field involving algebraic functions
    - Use Green's Theorem to evaluate a line integral on a region with holes through a vector field involving transcendental functions
- Applications of Green's Theorem
  - Use Green's Theorem to solve applications
    - Use Green's Theorem to calculate work along a closed rectangular path through a vector field involving algebraic functions
    - Use Green's Theorem to calculate work along a closed rectangular path through a vector field involving transcendental functions
    - Use Green's Theorem to calculate work along a closed circular path through a vector field involving algebraic functions
    - Use Green's Theorem to calculate work along a closed circular path through a vector field involving transcendental functions
    - Use Green's Theorem to calculate work along a closed triangular path through a vector field involving algebraic functions

- Use Green's Theorem to calculate work along a closed triangular path through a vector field involving transcendental functions
- Use Green's Theorem to calculate work along a closed elliptical path
- Use Green's Theorem to calculate work along a general closed path through a vector field involving algebraic functions
- Use Green's Theorem to calculate work along a general closed path through a vector field involving transcendental functions
- Use Green's Theorem to find the area of a region enclosed by a curve
- Use Green's Theorem to find the flux of a fluid through a region

16.5 Divergence and Curl

- Divergence
  - Find the divergence of a vector field
    - Find the divergence of a two-dimensional vector field involving polynomial functions
    - Find the divergence of a two-dimensional vector field involving radical functions
    - Find the divergence of a two-dimensional vector field involving trigonometric functions
    - Find the divergence of a three-dimensional vector field involving polynomial functions
    - Find the divergence of a three-dimensional vector field involving trigonometric functions
    - Find the divergence of a three-dimensional vector field involving exponential functions
  - Find the divergence of a field at a point
    - Find the divergence of a two-dimensional vector field involving polynomial functions at a point
    - Find the divergence of a two-dimensional vector field involving radical functions at a point
    - Find the divergence of a two-dimensional vector field involving trigonometric functions at a point
    - Find the divergence of a three-dimensional vector field involving polynomial functions at a point
    - Find the divergence of a three-dimensional vector field involving trigonometric functions at a point
    - Find the divergence of a three-dimensional vector field involving exponential functions at a point
  - Use divergence to determine whether a vector field is source free
    - Determine whether a vector field involving polynomials is source-free
    - Determine whether a vector field involving trigonometric functions is source-free
- Curl
  - Find the curl of a vector field
    - Find the curl of a three-dimensional vector field involving polynomial functions
    - Find the curl of a three-dimensional vector field involving trigonometric functions
    - Find the curl of a three-dimensional vector field involving exponential functions
    - Find the curl of a two-dimensional vector field involving polynomial functions

        - Find the curl of a two-dimensional vector field involving trigonometric functions
        - Find the curl of a two-dimensional vector field involving exponential functions
    - Find the curl of a vector field at a point
        - Find the curl of a three-dimensional vector field involving polynomial functions at a point
        - Find the curl of a three-dimensional vector field involving trigonometric functions at a point
        - Find the curl of a three-dimensional vector field involving exponential functions at a point
        - Find the curl of a two-dimensional vector field involving polynomial functions at a point
        - Find the curl of a two-dimensional vector field involving trigonometric functions at a point
        - Find the curl of a two-dimensional vector field involving exponential functions at a point
- Applications of Curl
    - Use the curl to determine whether a vector field is conservative
        - Determine whether a two-dimensional vector field involving polynomials is conservative
        - Determine whether a two-dimensional vector field involving trigonometric functions is conservative
        - Determine whether a three-dimensional vector field involving polynomials is conservative
        - Determine whether a three-dimensional vector field involving trigonometric functions is conservative
    - Determine whether a vector field could be the curl of another vector field
        - Determine whether a two-dimensional vector field involving polynomials could be the curl of another vector field
        - Determine whether a two-dimensional vector field involving trigonometric functions could be the curl of another vector field
        - Determine whether a three-dimensional vector field involving polynomials could be the curl of another vector field
    - Use the Laplace operator to determine whether a scalar function is harmonic
        - Determine whether a polynomial function of two variables is harmonic
        - Determine whether a transcendental function of two variables is harmonic
        - Determine whether a polynomial function of three variables is harmonic
        - Determine whether a transcendental function of three variables is harmonic
- Divergence and Flow
    - Use divergence to understand the flow of a fluid
        - Use divergence to find the flow of a fluid with a polynomial velocity field at a point in two dimensions
        - Use divergence to find the flow of a fluid with a trigonometric velocity field at a point in two dimensions
        - Use divergence to find the flow of a fluid with a polynomial velocity field at a point in three dimensions

        - Use divergence to find the flow of a fluid with a trigonometric velocity field at a point in three dimensions
        - Find a point at which the inflow and outflow of a fluid are equal in two dimensions
    - Use divergence to understand electromagnetic fields
        - Determine whether a two-dimensional vector field involving polynomials could be a magnetic field
        - Determine whether a two-dimensional vector field involving trigonometric functions could be a magnetic field
        - Determine whether a three-dimensional vector field could be a magnetic field
        - Determine whether a polynomial function in two variables could be the potential function of an electrostatic field
        - Determine whether a transcendental function in two variables could be the potential function of an electrostatic field
        - Determine whether a polynomial function in three variables could be the potential function of an electrostatic field
        - Determine whether a transcendental function in three variables could be the potential function of an electrostatic field

16.6 Surface Integrals

- Parametric Surfaces
    - Find a parameterization for a surface
        - Identify the surface described by a parameterization
        - Find a parameterization for a surface
        - Identify a smooth surface
    - Find the surface area of a parameterized surface
        - Set up a surface integral to find a surface area given a polynomial parameterization
        - Set up a surface integral to find a surface area given a trigonometric parameterization
        - Find a surface area given a polynomial parameterization
        - Find a surface area given a trigonometric parameterization
        - Find the surface area of a planar surface
        - Find the surface area of a cylindrical surface
        - Find the surface area of a spherical surface
    - Find the surface area of a surface of revolution
        - Set up a surface integral to find the surface area of a surface of revolution
        - Find the surface area of a surface of revolution
- Scalar Surface Integrals
    - Evaluate a surface integral of a scalar valued function
        - Evaluate a surface integral of a scalar valued function given a polynomial parameterization
        - Evaluate a surface integral of a scalar valued function given a trigonometric parameterization
        - Evaluate the surface integral of a scalar valued function over a planar surface
        - Evaluate the surface integral of a scalar valued function over a cylindrical surface

- Surface Integrals of a Vector Field
  - Understand the orientation of a surface
    - Find an orientation for a parametric surface
    - Find a tangent plane to a parametric surface
  - Evaluate a surface integral of a vector field
    - Evaluate a surface integral of a vector field given a polynomial parameterization
    - Evaluate a surface integral of a vector field given a trigonometric parameterization
    - Evaluate the surface integral of a vector field over a planar surface
    - Evaluate the surface integral of a vector field over a spherical surface
- Applications of Surface Integrals
  - Use Surface integrals to Solve Applications
    - Find the mass of a sheet described by a parameterized surface
    - Find a mass flow rate using a surface integral
    - Find a heat flow rate using a surface integral

16.7 Stokes' Theorem

- Understand and Apply Stokes' Theorem
  - Understand Stokes' theorem
    - Write a line integral that is equivalent to a given surface integral over a parabolic surface with circular boundary
    - Write a line integral that is equivalent to a given surface integral over a general surface with triangular boundary
    - Write a surface integral that is equivalent to a given line integral along a triangular path
    - Write a surface integral that is equivalent to a given line integral along an elliptic path
  - Use Stokes' theorem to evaluate a surface integral
    - Use Stokes' theorem to evaluate a surface integral over a planar surface with triangular boundary
    - Use Stokes' theorem to evaluate a surface integral over a planar surface with elliptic boundary
    - Use Stokes' theorem to evaluate a surface integral over a parabolic surface with semicircular boundary
    - Use Stokes' theorem to evaluate a surface integral over a hemispherical surface with circular boundary
    - Use Stokes' theorem to evaluate a surface integral over an elliptic surface with elliptic boundary
  - Use Stokes' theorem to evaluate a line integral
    - Use Stokes' theorem to evaluate a line integral along a triangular path
    - Use Stokes' theorem to evaluate a line integral along a circular path
    - Use Stokes' theorem to evaluate a line integral along an elliptic path
  - Use Stoke's theorem in applications
    - Use Stokes' theorem to find the circulation of a fluid along a triangular path
    - Use Stokes' theorem to find the circulation of a fluid along a circular path

- Use Faraday's law to compute the curl of an electric field given a constant magnetic field
- Use Faraday's law to compute the curl of an electric field given a uniform magnetic field that varies with time
- Use Faraday's law to compute the curl of an electric field given a radial magnetic field that varies with time

16.8 The Divergence Theorem

- Understand and Apply the Divergence Theorem
  - Understand the divergence theorem
    - Write a triple integral that is equivalent to a given flux integral across a closed rectangular surface
    - Write a triple integral that is equivalent to a given flux integral across a closed polyhedral surface
    - Write a triple integral that is equivalent to a given flux integral across a closed cylindrical surface
    - Write a triple integral that is equivalent to a given flux integral across a closed parabolic surface
    - Write a triple integral that is equivalent to a given flux integral across a closed conical surface
    - Write a triple integral that is equivalent to a given flux integral across a closed spherical surface
  - Use the divergence theorem to determine the flux of a vector field across a closed surface
    - Use the divergence theorem to determine the flux of an algebraic vector field across a closed rectangular surface
    - Use the divergence theorem to determine the flux of a transcendental vector field across a closed rectangular surface
    - Use the divergence theorem to determine the flux of an algebraic vector field across a closed polyhedral surface
    - Use the divergence theorem to determine the flux of an algebraic vector field across a closed cylindrical surface
    - Use the divergence theorem to determine the flux of a transcendental vector field across a closed cylindrical surface
    - Use the divergence theorem to determine the flux of an algebraic vector field across a closed parabolic surface
    - Use the divergence theorem to determine the flux of a transcendental vector field across a closed parabolic surface
    - Use the divergence theorem to determine the flux of an algebraic vector field across a closed conical surface
    - Use the divergence theorem to determine the flux of an algebraic vector field across a closed spherical surface
    - Use the divergence theorem to determine the flux of a transcendental vector field across a closed spherical surface
  - Use the divergence theorem in applications

- Use the divergence theorem to determine the flux of a fluid with a polynomial velocity field across a closed surface
- Use the divergence theorem to determine the flux of a fluid with an exponential velocity field across a closed surface
- Use the divergence theorem to determine the flux of an electrostatic field across a closed cylindrical surface
- Use the divergence theorem to determine the flux of an electrostatic field across a closed spherical surface
- Use the divergence theorem to determine the heat flux of a polynomial temperature distribution across a closed surface
- Use the divergence theorem to determine the heat flux of an exponential temperature distribution across a closed surface

Chapter 17: Second-Order Differential Equations

17.1 Second-Order Linear Equations

- Understanding and Classifying Linear Differential Equations
  - Understand and classify linear differential equations
    - Classify a differential equation as linear or nonlinear
    - Classify a linear differential equation as homogeneous or nonhomogeneous
    - Verify a solution to a differential equation
- The Superposition Principle
  - Apply the superposition principle to second-order homogeneous linear differential equations
    - Verify the superposition principle for homogeneous differential equations
    - Determine if a given pair of functions is linearly dependent or linearly independent
    - Find the general solution to a homogeneous differential equation given two linearly independent solutions
- Solving Homogeneous Second-Order Linear Differential Equations
  - Solve homogeneous second-order linear differential equations with constant coefficients
    - Determine the characteristic equation of a second-order linear differential equation
    - Determine the type and number of roots of the characteristic equation of a second-order linear differential equation
    - Solve a second-order linear differential equation for which the characteristic equation has two distinct real roots
    - Solve a second-order linear differential equation for which the characteristic equation has a repeated real root
    - Solve a second-order linear differential equation for which the characteristic equation has complex roots
    - Determine the second-order linear homogeneous differential equation with constant coefficients that has a given general solution
    - Solve a second-order linear differential equation
- Initial Value Problems for Second-Order Linear Differential Equations
  - Solve initial-value and boundary-value problems involving linear differential equations

- Solve an initial-value problem for a second-order differential equation
- Solve a second-order differential equation representing a spring problem
- Solve a boundary-value problem for a second-order differential equation

17.2 Nonhomogeneous Linear Equations

- Finding Solutions Using Undetermined Coefficients
  - Write the general solution to a nonhomogeneous differential equation
    - Given a particular solution that is a polynomial function, find the general solution to a nonhomogeneous differential equation
    - Given a particular solution that is an exponential function, find the general solution to a nonhomogeneous differential equation
    - Given a particular solution that is a trigonometric function, find the general solution to a nonhomogeneous differential equation
    - Given a particular solution that is a combination of polynomial, exponential, or trigonometric functions, find the general solution to a nonhomogeneous differential equation
  - Write the form for the particular solution for the method of undetermined coefficients
    - Given a nonhomogeneous differential equation, determine the form for the particular solution for the method of undetermined coefficients
  - Solve a nonhomogeneous differential equation using the method of undetermined coefficients
    - Use the method of undetermined coefficients to solve a nonhomogeneous differential equation when r(x) is a polynomial function
    - Use the method of undetermined coefficients to solve a nonhomogeneous differential equation when r(x) is an exponential function
    - Use the method of undetermined coefficients to solve a nonhomogeneous differential equation when r(x) is a trigonometric function
    - Use the method of undetermined coefficients to solve a nonhomogeneous differential equation when r(x) is a combination of polynomial, exponential or trigonometric functions
    - Use the method of undetermined coefficients to solve a nonhomogeneous differential equation when r(x) is a solution of the complementary equation
- Initial Value Problems for Nonhomogeneous Differential Equations
  - Solve an initial value problem for a nonhomogeneous differential equation
    - Given a particular solution, determine the unique solution satisfying an initial value problem for a nonhomogeneous differential equation.
    - Use the method of undetermined coefficients to solve an initial value problem when r(x) is a trigonometric function
    - Use the method of undetermined coefficients to solve an initial value problem when r(x) is combination of polynomial and exponential functions
    - Finding Solutions Using Variation of Parameters
  - Find a particular solution for a nonhomogeneous differential equation using the method of variation of parameters
    - Use Cramer's rule to solve a system of equations with variable coefficients
    - Use the method of variation of parameters to find a particular solution to a nonhomogeneous differential equation when r(x) is a polynomial function

- Use the method of variation of parameters to find a particular solution to a nonhomogeneous differential equation when r(x) is an exponential function
- Use the method of variation of parameters to find a particular solution to a nonhomogeneous differential equation when r(x) is a trigonometric function
- Use the method of variation of parameters to find a particular solution to a nonhomogeneous differential equation when r(x) is a combination of polynomial and exponential functions
- Use the method of variation of parameters to find a particular solution to a nonhomogeneous differential equation with nonconstant coefficients

- Find a general solution for a nonhomogeneous differential equation using the method of variation of parameters
  - Use the method of variation of parameters to find a general solution to a nonhomogeneous differential equation when r(x) is a polynomial function
  - Use the method of variation of parameters to find a general solution to a nonhomogeneous differential equation when r(x) is an exponential function
  - Use the method of variation of parameters to find a general solution to a nonhomogeneous differential equation when r(x) is a trigonometric function
  - Use the method of variation of parameters to find a general solution to a nonhomogeneous differential equation when r(x) is a combination of polynomial, exponential or trigonometric functions

17.3 Applications

- Simple Harmonic Motion
  - Solve second-order differential equations involving simple harmonic motion
    - Solve second-order differential equations involving simple harmonic motion
      Solve a second-order differential equation representing damped simple harmonic motion
    - Solve second-order differential equations involving simple harmonic motion
      Express a sum of trigonometric functions as a single trigonometric function
- Damped Spring-Mass Systems
  - Solve second-order differential equations involving damped simple harmonic motion
    - Solve a second-order differential equation representing an overdamped spring-mass system
    - Solve a second-order differential equation representing a critically damped spring-mass system
    - Solve a second-order differential equation representing an underdamped spring-mass system
  - Solve second-order differential equations involving forced simple harmonic motion
    - Find the steady-state solution to a second-order differential equation representing forced simple harmonic motion
    - Find the transient solution to a second-order differential equation representing forced simple harmonic motion
- RLC Series Circuits
  - Solve second-order differential equations involving an RLC circuit

- Find the charge on the capacitor in an RLC series circuit represented by a second-order differential equation
- Find the current in an RLC series circuit represented by a second-order differential equation

17.4 Series Solutions of Differential Equations

- Solve Differential Equations with Power Series
  - Use power series to solve a first-order differential equation
    - Find a power series solution to a first-order differential equation with constant coefficients
    - Find a power series solution to a first-order differential equation with nonconstant coefficients
  - Use power series to solve a second-order differential equation
    - Find a power series solution to a second-order differential equation with constant coefficients
    - Find a power series solution to a second-order differential equation with nonconstant coefficients
    - Find a power series solution to a second-order initial-value problem with constant coefficients
    - Find a power series solution to a second-order initial-value problem with nonconstant coefficients