

Calculus with Precalculus Review | Table of Contents

Chapter 1: Functions and Graphs

1.1 Review of Functions

- Functions
 - Determine the domain and range of a relation and if a given relation is a function
 - Determine the domain and range of a relation given discrete values
 - Determine the domain and range of a relation given a graph
 - Determine if a relation is a function given discrete values
 - Determine if a relation is a function given a graph
 - Use functional notation to evaluate a function
 - Evaluate a function with a numerical argument
 - Evaluate a function with a variable argument
 - Determine the domain and range of a function
 - Determine the domain and range of a polynomial function
 - Determine the domain and range of a radical function
 - Determine the domain and range of a rational function
 - Representing Functions
 - Find the zeros and intercepts of a function
 - Determine the zeros and intercepts of a linear function
 - Determine the zeros and intercepts of a rational function
 - Determine the zeros and intercepts of a radical function
 - Determine the zeros and intercepts of a function given a graph
 - Use a table to sketch a function
 - Complete a table of function values given a function
 - Sketch a function given a table of values
 - Determine the intervals where a function is increasing or decreasing
 - Determine the intervals where a function is increasing or decreasing given a function
 - Determine the intervals where a function is increasing or decreasing from a graph
 - Combining Functions and Symmetry of Functions
 - Combine functions using mathematical operations
 - Combine functions using addition or subtraction
 - Combine functions using multiplication
 - Combine functions using division
 - Determine the domain of a combination of functions
 - Perform a composition of functions
 - Perform a composition of functions defined by formulas
-

- Perform a composition of functions defined by formulas at a point
- Determine the domain of a composition of functions defined by formulas
- Perform a composition of functions defined by a table
- Describe the symmetry properties of a function
 - Determine if a function is symmetric about an axis or the origin from a graph
 - Determine if a function is even or odd from a graph
 - Determine if a function is even or odd from a function

1.2 Basic Classes of Functions

- Linear Functions and Slope
 - Calculate the slope of a linear function
 - Calculate the slope given two points
 - Calculate the slope given a graph
 - Calculate the slope given a linear function
 - Find the equation of a line
 - Write the equation of a line in point-slope form given the slope and a point
 - Write the equation of a line in point-slope form given two points
 - Write the equation of a line in slope-intercept form given the slope and a point
 - Write the equation of a line in slope-intercept form given two points
 - Write the equation of a line given the x- and y-intercepts
 - Graph a linear equation
 - Identify the slope and intercept given a line in slope-intercept form
 - Graph a linear equation given in slope-intercept form
 - Use a linear function to model an application
 - Write a linear function to model an application
 - Interpret a linear function given an application
 - Graph a linear function to model an application
 - Polynomials
 - Recognize key components of a polynomial
 - Recognize the degree of a polynomial
 - Find the roots of a factorable polynomial
 - Find the roots of a quadratic function using the quadratic formula
 - Describe the end-behavior of a polynomial function
 - Graph a polynomial function
 - Graph a polynomial function given information about the function
 - Graph a polynomial function
 - Use polynomials to model an application
 - Interpret a polynomial function given an application
 - Graph a polynomial function to model an application
 - Determine whether a function is algebraic or transcendental
-

- Piecewise Functions and Transformations of Functions
 - Use piecewise functions
 - Evaluate a piecewise function
 - Graph a piecewise defined function
 - Write a piecewise function to model an application
 - Perform a transformation of functions
 - Transform a function by performing a horizontal or vertical shift
 - Transform a function by performing a vertical or horizontal compression
 - Transform a function by performing a reflection about the x-axis or y-axis

1.3 Trigonometric Functions

- Radian Measure and the Trigonometric Functions
 - Understand the unit circle and convert angle measures between degrees and radians
 - Convert an angle measure from degrees to radians
 - Convert an angle measure from radians to degrees
 - Find a coterminal angle
 - Find the reference angle for a given angle
 - Use the circular definitions of the basic trigonometric functions
 - Find the exact value of a sine or cosine function given an angle measure
 - Find the exact value of a tangent or cotangent function given an angle measure
 - Find the exact value of a secant or cosecant function given an angle measure
 - Relate points on the unit circle to a trigonometric ratio
 - Use the triangular definitions of the basic trigonometric functions
 - Find a missing side length of a right triangle given two sides
 - Find a trigonometric ratio given side lengths of a right triangle
 - Find a missing side length of a right triangle given a side and an angle
 - Trigonometric Equations and Identities
 - Solve trigonometric equations
 - Solve a trigonometric equation involving sine or cosine
 - Solve a trigonometric equation involving tangent or cotangent
 - Solve a trigonometric equation involving secant or cosecant
 - Solve an application problem involving trigonometric equations
 - Use trigonometric identities to simplify and rewrite expressions
 - Use the reciprocal trigonometric identities to write an expression in terms of sine and cosine
 - Use the Pythagorean trigonometric identities to simplify an expression
 - Use the addition and subtraction formulas to rewrite an expression
 - Use the double-angle formulas to simplify an expression
 - Graphing Trigonometric Functions
 - Graph trigonometric functions
-

- Graph a trigonometric equation involving sine or cosine
- Graph a trigonometric equation involving tangent or cotangent
- Graph a trigonometric equation involving secant or cosecant

1.4 Inverse Functions

- Finding Inverse Functions
 - Determine if a function is one-to-one
 - Determine if a function is one-to-one given a set of ordered pairs
 - Determine if a function is one-to-one given a table of values
 - Determine if a function is one-to-one given a graph using the horizontal line test
 - Find the inverse of a function
 - Determine if two functions are inverses of each other given a graph
 - Graph the inverse of a function
 - Determine if two functions are inverses of each other by performing a composition of functions
 - Find the inverse of a function algebraically
 - Understand domain and range of an inverse function
 - Find the domain and range of the inverse of a polynomial function
 - Find the domain and range of the inverse of a radical function
 - Find the domain and range of the inverse of a rational function
 - Restrict the domain of a function to find an inverse function
- Inverse Trigonometric Functions
 - Evaluate inverse trigonometric functions
 - Determine the domain of inverse trigonometric functions
 - Identify the graphs of inverse trigonometric functions
 - Evaluate inverse trigonometric functions
 - Use inverse trigonometric functions to solve trigonometric equations

1.5 Exponential and Logarithmic Functions

- Exponential Functions
 - Evaluate an exponential function
 - Evaluate an exponential function at an integer value
 - Evaluate an exponential function at a rational value
 - Evaluate an exponential function in an application
 - Use the properties of exponents to simplify expressions
 - Use the properties of exponents to simplify expressions with a single variable
 - Use the properties of exponents to simplify expressions with a single variable and rational exponents
 - Use the properties of exponents to simplify algebraic expressions with two variables
 - Relate an exponential function to its graph
-

- Determine the domain, range, and asymptotes of an exponential function
 - Graph an exponential function
 - Graph an exponential function using transformations
 - Find the equation for an exponential function given its graph
 - Use exponential functions and base e in applications
 - Write an exponential function to model an application
 - Interpret an exponential function given an application
 - Graph an exponential function to model an application
 - Logarithmic Functions
 - Relate a logarithmic expression to an exponential expression and use properties of logarithms
 - Write an exponential expression in logarithmic form
 - Write a logarithmic expression in exponential form
 - Use the properties of logarithms to condense a logarithmic expression
 - Use the properties of logarithms to expand a logarithmic expression
 - Evaluate a logarithmic expression
 - Relate a logarithmic function to its graph
 - Determine the domain, range, and asymptotes of a logarithmic function
 - Graph a logarithmic function
 - Graph a logarithmic function using transformations
 - Find the equation for a logarithmic function given its graph
 - Exponential and Logarithmic Equations
 - Solve exponential and logarithmic equations
 - Solve a factorable exponential equation
 - Solve an exponential equation using logarithms
 - Solve a logarithmic equation by exponentiating
 - Solve a logarithmic equation by using the properties of logarithms
 - Hyperbolic Functions
 - Evaluate a hyperbolic function
 - Evaluate a hyperbolic function involving hyperbolic sine or cosine
 - Evaluate a hyperbolic function involving hyperbolic tangent or cotangent
 - Evaluate a hyperbolic function involving hyperbolic secant or cosecant
 - Given the value of one hyperbolic function find the value of another
 - Graph a hyperbolic function
 - Graph a hyperbolic function involving hyperbolic sine or cosine
 - Graph a hyperbolic function involving hyperbolic tangent or cotangent
 - Graph a hyperbolic function involving hyperbolic secant or cosecant
 - Inverse Hyperbolic Functions
 - Evaluate an inverse hyperbolic function
-

- Evaluate an inverse hyperbolic function involving inverse hyperbolic sine or cosine
- Evaluate an inverse hyperbolic function involving inverse hyperbolic tangent or cotangent
- Evaluate an inverse hyperbolic function involving inverse hyperbolic secant or cosecant
- Graph an inverse hyperbolic function
 - Graph an inverse hyperbolic function involving inverse hyperbolic sine or cosine
 - Graph an inverse hyperbolic function involving inverse hyperbolic tangent or cotangent
 - Graph an inverse hyperbolic function involving inverse hyperbolic secant or cosecant

Chapter 2: Limits

2.1 A Preview of Calculus

- A Preview of Differential and Integral Calculus
 - Preview the area problem by using rectangles to approximate area
 - Find the slope of secant lines
 - Estimate the slope of the tangent line to a point using secant lines
 - Find the average velocity
 - Estimate the instantaneous velocity at a given time using average velocities
 - Preview the area problem by using rectangles to approximate area

2.2 The Limit of a Function

- The Intuitive Definition of a Limit
 - Understand the limit of a function and evaluate a limit from a table
 - Use correct notation to describe the limit of a function
 - Use a table of values to estimate the limit of a function or to identify when the limit does not exist
 - Evaluate limits graphically
 - One-Sided Limits
 - Evaluate a one-sided limit from a table or a graph
 - Evaluate a one-sided limit from a table
 - Evaluate a one-sided limit from a graph
 - Explain the relationship between one-sided and two-sided limits
 - Determine when a two-sided limit exists by analyzing one-sided limits from a table
 - Determine when a two-sided limit exists by analyzing one-sided limits from a graph
 - Infinite Limits
-

- Find an infinite limit
 - Use correct notation to describe the infinite limit of a function
 - Determine an infinite limit from a table
 - Determine an infinite limit from a graph
- Define a vertical asymptote

2.3 The Limit Laws

- Evaluating Limits with the Limit Laws
 - Use the limit laws to evaluate the limit of a function
 - Use the sum, difference, or constant multiple law for limits to evaluate a limit
 - Use the quotient or power law for limits to evaluate a limit
 - Use the product or root law for limits to evaluate a limit
- Evaluate a Basic Limit
 - Find the limit of a polynomial or rational function using limit laws and direct substitution
 - Find the limit of a polynomial function
 - Find the limit of a rational function
 - Use one-sided limits to evaluate a limit using limit laws
 - Evaluate a one-sided limit using the limit laws
 - Find the limit of a piecewise function
 - Find the limit of an absolute value function
 - Find the limit of a step function
- Additional Limit Evaluation Techniques
 - Evaluate the limit of a rational function by simplification
 - Find the limit of a function with removable discontinuities by factoring
 - Find the limit of a function with removable discontinuities through expansion
 - Find the limit of a function by adding two rational expressions
 - Evaluate the limit of a function with complex fractions or rational functions that contain radicals
 - Find the limit of a rational function that contains radicals
 - Find the limit of a function with complex fractions
- The Squeeze Theorem and Limits of Trigonometric Functions
 - Evaluate the limit of a function by using the squeeze theorem
 - Demonstrate a conceptual understanding of the squeeze theorem
 - Use the squeeze theorem to find the limit of a trigonometric function
 - Evaluate the limit of a trigonometric function by direct substitution
 - Evaluate the limit of a trigonometric function using special limits
 - Evaluate trigonometric limits using special limits
 - Evaluate trigonometric limits using special limits and trigonometric identities

2.4 Continuity

- Continuity at a Point
-

- Explain the three conditions for continuity at a point
 - Determine which condition of the definition of continuity fails given a graph
 - Determine which condition of the definition of continuity fails given a function
- Distinguish between three kinds of discontinuity
 - Distinguish between types of discontinuity given a graph
 - Distinguish between types of discontinuity given a function
- Determine the continuity of a piecewise function
 - Determine whether a piecewise function is continuous at a given value
 - Determine the value that makes a piecewise function continuous
 - Determine the intervals on which a piecewise function is continuous from a graph
- Continuity over an Interval and the Intermediate Value Theorem
 - Determine where an algebraic function is continuous
 - Determine where a polynomial or rational function is continuous
 - Determine where a radical function is continuous
 - Determine the limit of a composite function using the composite function theorem
 - Determine the limit of a composite trigonometric function
 - Determine the limit of a composite exponential or logarithmic function
 - Determine the limit of a composite inverse trigonometric function
 - Understand and apply the Intermediate Value Theorem
 - Determine if the intermediate value theorem applies
 - Determine how many roots a continuous function may have given a table
 - Use the intermediate value theorem given an equation

2.5 The Precise Definition of a Limit

- Use the Epsilon-Delta Definition of the Limit to Prove Statements about Limits
 - Understand the notation in finding the formal definition of a limit
 - Given a limit statement, write the corresponding epsilon-delta statement
 - Given the epsilon-delta statement of a limit, write the corresponding limit statement
 - Use graphs to demonstrate an understanding of the epsilon-delta definition of limits
 - Determine a delta for a given or arbitrary epsilon given a function
 - Determine a delta for a given epsilon for a linear function
 - Determine a delta for an arbitrary epsilon given a linear function
 - Determine a delta for an arbitrary epsilon given a quadratic function
 - Use the epsilon-delta definition of limits to understand one-sided and infinite limits
 - Prove a statement about a limit from one side
 - Prove a statement about infinite limits
-

Chapter 3: Derivatives

3.1 Defining the Derivative

- Tangent Lines
 - Find the slope of a secant line
 - Find the slope of the secant line given a function
 - Find the slope of the secant line given a table or graph
 - Find the slope of the secant line given a function and variable intervals
 - Find the slope of a tangent line using the first limit definition
 - Find the slope of a tangent line given a linear function using the first limit definition
 - Find the slope of a tangent line given a quadratic function using the first limit definition and factoring
 - Find the slope of a tangent line given a rational function using the first limit definition and complex fractions
 - Find the slope of a tangent line given a radical function using the first limit definition
 - Find the slope of a tangent line using the second limit definition
 - Find the slope of a tangent line given a linear function using the second limit definition
 - Find the slope of a tangent line given a quadratic function using the second limit definition and expansion
 - Find the slope of a tangent line given a rational function using the second limit definition and complex fractions
 - Find the slope of a tangent line given a radical function using the second limit definition
 - Find the equation of a tangent line using a limit definition
 - Find the equation of a tangent line given a linear function using the second limit definition
 - Find the equation of a tangent line given a quadratic function using the second limit definition and expansion
 - Find the equation of a tangent line given a rational function using the second limit definition and complex fractions
 - Find the equation of a tangent line given a radical function using the second limit definition
 - The Derivative of a Function at a Point
 - Identify the derivative as the limit of a difference quotient
 - Identify the function f and number a given the limit definition of a derivative for a polynomial function
-

- Identify the function f and number a given the limit definition of a derivative for a radical function
- Identify the function f and number a given the limit definition of a derivative for a rational function
- Write the limit definition of a derivative given a function and a value
- Estimate the value of a derivative at a point
- Calculate the derivative of a given function at a point using a limit
 - Find the value of the derivative at a point given a linear function using the definition of a derivative
 - Find the value of the derivative at a point given a quadratic function using the definition of a derivative
 - Find the value of the derivative at a point given a rational function using the definition of a derivative
 - Find the value of the derivative at a point given a radical function using the definition of a derivative
- Velocities and Rates of Change
 - Use the concept of a derivative to find an instantaneous velocity
 - Estimate the velocity at a time given a table of values
 - Use the limit definition of a derivative to find the instantaneous velocity given a position function
 - Use instantaneous rates of change in other contexts
 - Estimate the instantaneous rate of change in different contexts given a table of values
 - Determine the rate of change of temperature given a function
 - Determine the rate of change in a business context given a function

3.2 The Derivative as a Function

- Derivative Functions and their Graphs
 - Define the derivative function of a given function
 - Find the derivative function of a linear or quadratic function using limits
 - Find the derivative function of a radical function using limits
 - Find the derivative function of a rational function using limits
 - Graph the derivative of a function
 - Estimate the value of a derivative at a point on a graph using a tangent line
 - Determine the open intervals where the derivative is positive or negative from a graph
 - Identify the graph of a derivative function given a function
 - Graph a derivative function given a function
 - Derivatives and Continuity
 - Understand differentiability and when a function does not have a derivative
-

- State the connection between derivatives and continuity
- Describe three conditions for when a function does not have a derivative
- Determine the differentiability of piecewise functions
 - Determine if a piecewise function is differentiable
 - Determine if a piecewise function is differentiable given a graph
 - Find the constants that make a piecewise function differentiable
- Higher-Order Derivatives
 - Determine the second derivative of a function using the limit definition
 - Interpret the meaning of a higher-order derivative
 - Find the second derivative of a polynomial function
 - Graph higher-order derivative functions
 - Identify the graph of a higher-order derivative function given a function
 - Graph a higher-order derivative function given a function
 - Relate higher-order derivatives to acceleration
 - Interpret the meaning of a higher-order derivative with respect to a position function
 - Find the acceleration function given a position function using limits

3.3 Differentiation Rules

- The Basic Differentiation Rules
 - Use the constant, constant multiple, and power rules
 - Find the derivative of a constant function
 - Find the derivative of a function using the power rule for monomials
 - Find the derivative of a function using the constant multiple rule with an integer coefficient
 - Find the derivative of a function using the constant multiple rule with a fractional coefficient
 - Apply the sum and difference rules to combine derivatives
 - Find the derivative of a function using the sum of functions
 - Find the derivative of a function using the difference of two functions
 - Find the derivative of a function using both the sum and difference of functions
 - Find the derivative of a function using both the sum and difference of functions with fractional coefficients
 - Evaluate the derivative of a function at a point using both the sum and difference of functions with fractional coefficients
 - Find the derivative of a function using sum and difference rules and simplification
 - Find the equation of a tangent line using basic derivative rules
 - Find the equation of a line tangent to a monomial function
 - Find the equation of a line tangent to a polynomial function
 - Graph a line tangent to a polynomial function
-

- Find the slope of the line tangent to a polynomial function
- Find the equation of a line normal to a polynomial function
- The Product and Quotient Rules
 - Use the product rule to find the derivative of a product of functions
 - Apply the product rule given information of two functions at a point
 - Apply the product rule given one function and information of another function at a point
 - Find the derivative of the product of two functions
 - Find the value of the derivative at a point of the product of two functions
 - Use the quotient rule to find the derivative of a quotient of functions
 - Apply the quotient rule given information of two functions at a point
 - Apply the quotient rule given one function and information of another function at a point
 - Find the derivative of the quotient of two functions
 - Find the value of the derivative at a point of the quotient of two functions
 - Combine the product and quotient rules
 - Combine the product or quotient rule with the sum and difference rules
 - Find the derivative of the product of three functions
 - Combine the product and the quotient rule
- Extensions of the Power Rule
 - Extend the power rule to functions with negative exponents
 - Find the derivative of a monomial function with negative exponents
 - Find the derivative of a function with negative exponents
 - Rewrite a rational function to use the power rule to find its derivative
 - Extend the power rule to functions with rational exponents
 - Find the derivative of a monomial function with rational exponents
 - Find the derivative of a function with rational exponents
 - Find the derivative of a function with negative rational exponents
 - Rewrite a function with radicals to use the power rule to find its derivative
 - Rewrite a function with radicals to use the power rule and simplify to find its derivative
 - Use the extensions of the power rule to find tangent lines
 - Find the value of the derivative of a point using the extension of the power rule
 - Find the equation of the line tangent to a point using the extension of the power rule
 - Graph the equation of the line tangent to a point using the extension of the power rule

3.4 Derivatives as Rates of Change

- Motion along a Line
-

- Find the time that a moving object has a certain position given the position function
- Find the velocity function given the position function
 - Find the velocity function given the position function
 - Interpret the movement of an object given its position function
- Find the acceleration function given a position function
- Interpret the relationship between the velocity function and acceleration function
- Changes in Cost and Revenue
 - Use cost, revenue, and profit functions
 - Find the cost of producing a given number of items given a cost function
 - Create a revenue function given a price function
 - Create a profit function given a price function and cost function
 - Use derivatives to calculate marginal cost and revenue in a business situation
 - Find a marginal cost function
 - Find a marginal revenue function
 - Find a marginal profit function

3.5 Derivatives of Trigonometric Functions

- Derivatives of Trigonometric Functions
 - Find the derivative of a sine or cosine function
 - Find the derivative of a sine or cosine function using sum, difference, or constant multiple rules
 - Find the derivative of a sine or cosine function combined with the product rule
 - Find the derivative of a sine or cosine function combined with the quotient rule
 - Find the derivative of other trigonometric functions
 - Find the derivative of a tangent or cotangent function using sum, difference, or constant multiple rules
 - Find the derivative of a secant or cosecant function using sum, difference, or constant multiple rules
 - Find the derivative of a combination of trigonometric functions using sum, difference, or constant multiple rules
 - Find the derivative of a combination of trigonometric functions using the product rule
 - Find the derivative of a combination of trigonometric functions using the quotient rule
 - Find a higher-order derivative of a sine or cosine function
 - Find a higher-order derivative of a sine function
 - Find a higher-order derivative of a cosine function
 - Find the line tangent to a trigonometric function
 - Find the line tangent to a sine or cosine function
 - Find the line tangent to a tangent or cotangent function
-

- Find the line tangent to a secant or cosecant function

3.6 The Chain Rule

- The Chain Rule
 - Use the chain rule with the power rule
 - Decompose a composite function
 - Use the chain rule given decomposed functions
 - Use the chain rule given a graph
 - Use the chain rule with a polynomial function to some whole number power
 - Use the chain rule with a polynomial function to some negative integer power
 - Use the chain rule with a polynomial function to some rational power
 - Use the chain rule given functional values
 - Use the chain rule with the product or quotient rule
 - Use the chain rule with a polynomial function and the product rule
 - Use the chain rule with a polynomial function and the product rule with a radical
 - Use the chain rule with a polynomial function and the quotient rule
 - Use the chain rule with a polynomial function and the quotient rule with a radical
 - Use the chain rule with trigonometric functions
 - Use the chain rule with a sine or cosine function to some power
 - Use the chain rule with a sine or cosine function of some power
 - Differentiate a composite of three functions involving sine or cosine
 - Use the chain rule with a tangent or cotangent function
 - Use the chain rule with a secant or cosecant function

3.7 Derivatives of Inverse Functions

- Find the Derivative of an Inverse Function
 - Estimate the value of the derivative of an inverse function given the graph of a function
 - Calculate the derivative of an inverse function
 - Find an inverse derivative function value for a polynomial function
 - Find an inverse derivative function value for a rational function
 - Find an inverse derivative function for a radical function
 - Find the equation of the line tangent to an inverse function
 - Find an inverse derivative function value for a polynomial function
 - Graph the equation of the line tangent to the inverse of a polynomial function
 - Find an inverse derivative function value for a rational function
 - Find an inverse derivative function for a radical function
 - Find the Derivative of an Inverse Trigonometric Function
 - Write the algebraic form of an inverse trigonometric function
 - Find the derivative of an inverse trigonometric function
 - Find the derivative of an inverse trigonometric function at a value
 - Find the derivative of an inverse sine or cosine function using the chain rule
-

- Find the derivative of other inverse trigonometric functions using the chain rule

3.8 Implicit Differentiation

- Implicit Differentiation
 - Find the derivative of a function using implicit differentiation
 - Find the derivative of a function using implicit differentiation and the power rule
 - Find the derivative of a function using implicit differentiation and the extension of the power rule
 - Find the derivative of a function using implicit differentiation and the product rule
 - Find the derivative of a function using implicit differentiation and the product rule
 - Find the derivative of a function using implicit differentiation and the product rule with a sine or cosine expression
 - Use implicit differentiation to find a second derivative
 - Use implicit differentiation to find the equation of a tangent line
 - Find the value of the derivative at a point using implicit differentiation and the power rule
 - Find the value of the derivative at a point using implicit differentiation and the product rule
 - Find the equation of the tangent line at a point using implicit differentiation and the power rule
 - Find the equation of the tangent line at a point using implicit differentiation and the product rule

3.9 Derivatives of Exponential and Logarithmic Functions

- Derivatives of Exponential and Logarithmic Functions with Base e
 - Find the derivative of an exponential function with base e
 - Find the derivative of an exponential function with the product rule
 - Find the derivative of an exponential function with the chain rule
 - Find the derivative of an exponential function with the product rule and the chain rule
 - Find the derivative of an exponential function with the quotient rule
 - Find the derivative of an exponential function with a trigonometric function
 - Find the derivative of a natural logarithmic function
 - Find the derivative of a natural logarithmic function with the product rule
 - Find the derivative of a natural logarithmic function with the chain rule
 - Find the derivative of a natural logarithmic function with the product rule and the chain rule
 - Find the derivative of a natural logarithmic function with the quotient rule
 - Find the derivative of a natural logarithmic function with a trigonometric function
 - Find the derivative of a natural logarithmic function using properties of logarithms
-

- Find the derivative of a natural logarithmic function using properties of logarithms
- Find the derivative of a natural logarithmic function with a trigonometric function using properties of logarithms
- Derivatives of Exponential and Logarithmic Functions of Any Base
 - Find the derivative of an exponential function of any base
 - Find the derivative of an exponential function of any base with the product rule
 - Find the derivative of an exponential function of any base with the chain rule
 - Find the derivative of an exponential function of any base with the product and chain rules
 - Find the derivative of an exponential function of any base with the quotient rule
 - Find the derivative of an exponential function of any base with a trigonometric function
 - Find the derivative of a logarithmic function of any base
 - Find the derivative of a logarithmic function of any base with the chain rule
 - Find the derivative of a logarithmic function of any base with the product rule
 - Find the derivative of a logarithmic function of any base with the quotient rule
 - Find the derivative of a logarithmic function of any base with a trigonometric function
 - Find the derivative of a logarithmic function of any base using properties of logarithms
 - Find the derivative of a logarithmic function of any base using properties of logarithms
 - Find the derivative of a logarithmic function of any base with a trigonometric function using properties of logarithms
- Logarithmic Differentiation
 - Use logarithmic differentiation to find a derivative
 - Use logarithmic differentiation and the power property of logarithms to find the derivative of a function
 - Use logarithmic differentiation and the power property of logarithms to find the derivative of a function with a trigonometric function
 - Use logarithmic differentiation and properties of logarithms to find a derivative
 - Use logarithmic differentiation and all of the properties of logarithms to find the derivative of a function
 - Use logarithmic differentiation and all of the properties of logarithms to find the derivative of a function with a trigonometric function

Chapter 4: Applications of Derivatives

4.1 Related Rates

- Introduction to Related Rates and Related Rates with Volume or Area Problems
-

- Express changing quantities in terms of derivatives
- Use related rates given an abstract equation
- Use related rates to solve problems involving volume
 - Use related rates to solve problems involving volume
 - Use related rates to solve problems involving volume and similar figures
- Use related rates to solve problems involving area
 - Use related rates to solve problems involving area
 - Use related rates to solve problems involving area given two rates of change
- Related Rates in Other Applications
 - Use related rates to solve problems involving distance
 - Use related rates to solve problems involving distance with one length staying constant
 - Use related rates to solve problems involving distance with all lengths changing
 - Use related rates to solve problems involving angles or shadows
 - Use related rates to solve problems involving changing angles
 - Use related rates to solve problems involving shadows

4.2 Linear Approximations and Differentials

- Linear Approximation of a Function at a Point
 - Describe the linear approximation to a function at a point
 - Graphically represent a linear approximation
 - Find the linear approximation for a polynomial function
 - Find the linear approximation for a radical function
 - Find the linear approximation for a rational function
 - Find the linear approximation for a sine or cosine function
 - Use a linear approximation to estimate a function value
 - Use a linear approximation to estimate a polynomial function value
 - Use a linear approximation to estimate a radical function value
 - Use a linear approximation to estimate a rational function value
 - Use a linear approximation to estimate a sine or cosine function value
 - Differentials and Calculating Error
 - Find the differential of a function
 - Find the differential for a polynomial function
 - Find the differential for a radical function
 - Find the differential for a rational function
 - Find the differential for a sine or cosine function
 - Compute a differential
 - Compute a differential for a polynomial function
 - Compute a differential for a radical function
 - Compute a differential for a rational function
-

- Compute a differential for a sine or cosine function
- Represent a differential graphically
 - Represent a differential graphically
 - Represent a differential graphically and interpret its values
- Use a differential to find a change in a geometric context
 - Use a differential to find a change in volume
 - Use a differential to find a change in surface area
- Calculate the relative error and percentage error in using a differential approximation
 - Estimate the propagated error using differentials
 - Find the relative or percentage error using differentials

4.3 Maxima and Minima

- Critical Points and Extrema
 - Define absolute extrema and understand the extreme value theorem
 - Determine if the extreme value theorem applies for a given algebraic function
 - Determine if the extreme value theorem applies for a given trigonometric function
 - Determine if the extreme value theorem applies for a given exponential or logarithmic function
 - Determine from a graph if a function will have absolute extrema
 - Use derivatives to find critical points of a function
 - Identify local extrema from a graph
 - Find any critical points of a polynomial function
 - Find any critical points of a function with rational exponents
 - Find any critical points of a rational function
 - Find any critical points of a sine or cosine function
 - Find absolute extrema
 - Identify absolute extrema on a closed interval given a polynomial function
 - Identify absolute extrema on a closed interval given a function with rational exponents
 - Identify absolute extrema on a closed interval given a rational function
 - Identify absolute extrema on a closed interval given a sine or cosine function

4.4 The Mean Value Theorem

- Rolle's Theorem and the Mean Value Theorem for Derivatives
 - Apply Rolle's theorem
 - Determine if Rolle's theorem can be used for a function on a given interval
 - Represent Rolle's theorem graphically
 - Find all values c on a given interval that satisfy the conclusion of Rolle's theorem
 - Find values of c guaranteed by the Mean Value Theorem
 - Represent the Mean Value Theorem graphically
-

- Find values of c guaranteed by the Mean Value Theorem for a polynomial function
- Find values of c guaranteed by the Mean Value Theorem for a function with rational exponents
- Find values of c guaranteed by the Mean Value Theorem for a rational function
- Use the Mean Value Theorem and its corollaries to solve problems
- Use the Mean Value Theorem in applications

4.5 Derivatives and the Shape of a Graph

- The First Derivative Test
 - Understand the relationship between the graph of a function and the sign of its first derivative
 - Determine the sign of a derivative function given a graph of a function
 - Determine when a function is increasing or decreasing given the graph of its derivative
 - Determine when a function is increasing or decreasing given information about its derivative
 - Use the first derivative test to find local extrema of a function given its graph
 - Use the first derivative test to find local extrema of an algebraic function
 - Use the first derivative test and information about a function to find local extrema
 - Use the first derivative test to find local extrema of a polynomial function
 - Use the first derivative test to find local extrema of a function with rational exponents or radicals
 - Use the first derivative test to find local extrema of a function with a rational function
 - Use the first derivative test to find local extrema of a transcendental function
 - Use the first derivative test to find local extrema of a sine or cosine function
 - Use the first derivative test to find local extrema of an exponential function
 - Use the first derivative test to find local extrema of a logarithmic function
 - Concavity and Points of Inflection
 - Understand the relationship between the graph of a function and the sign of its second derivative
 - Determine the concavity of a function given its graph
 - Determine the concavity of a function given the graph of its second derivative
 - Determine the concavity of a function given the graph of its first derivative
 - Find inflection points and intervals of concavity given a function
 - The Second Derivative Test
 - Use the second derivative test to find local extrema of an algebraic function
-

- Use the second derivative test and information about a function to find local extrema
- Use the second derivative test to find local extrema of a polynomial function
- Use the second derivative test to find local extrema of a function with rational exponents or radicals
- Use the second derivative test to find local extrema of a transcendental function
 - Use the second derivative test to find local extrema of a sine or cosine function
 - Use the second derivative test to find local extrema of an exponential function
 - Use the second derivative test to find local extrema of a logarithmic function
- Use both the first and second derivatives of a function to determine the basic shape of a graph

4.6 Limits at Infinity and Asymptotes

- Limits at Infinity
 - Understand the meaning of a limit at infinity and horizontal asymptotes
 - Determine the limit at infinity of a function given the graph of a function
 - Determine the limit at infinity of a function using a table of values
 - Find the equation of the horizontal asymptote for a given function
 - Determine the limit at infinity of a polynomial function
 - Determine the limit at infinity of a power function
 - Determine the limit at infinity of a polynomial function
 - Determine the limit at infinity of a rational or other algebraic function
 - Determine the limit at infinity of a rational function
 - Determine the limit at infinity of a rational function involving a radical
 - Determine the limit at infinity of a function involving a radical
 - Determine the limit at infinity of a transcendental function
 - Determine the limit at infinity of a sine or cosine function
 - Determine the limit at infinity of another trigonometric function
 - Determine the limit at infinity of an exponential function
 - Determine the limit at infinity of a logarithmic function
 - Determine the limit at infinity of an inverse trigonometric function
 - Use the precise definition of a limit at infinity
 - Use the precise definition of a limit at infinity to write a statement proving a limit
 - Represent the precise definition of a limit at infinity graphically
 - Oblique Asymptotes and Curve Sketching
 - Find oblique asymptotes of a function
 - Determine when a function will have an oblique asymptote
 - Find the equation of the oblique asymptote for a given function
 - Graph the oblique asymptote of a function
 - Sketch the graph of a function
-

- Draw a graph of a polynomial
- Draw a graph of a rational function
- Draw a graph of a rational function with an oblique asymptote
- Draw a graph of a function with a cusp

4.7 Applied Optimization Problems

- Optimization Problems in a Geometric Context
 - Solve optimization problems involving area or volume
 - Solve an optimization problem involving area
 - Solve an optimization problem involving volume
 - Minimize surface area
 - Minimize distance or travel time
 - Minimize travel time using optimization
 - Minimize a distance using optimization
- Optimization Problems in Other Contexts
 - Solve optimization problems in the abstract
 - Minimize the distance from a function to a point
 - Maximize the area of an inscribed rectangle
 - Maximize or minimize quantities given an expression with two variables
 - Solve optimization problems in a business or economics context
 - Use a demand function to maximize revenue
 - Maximize profit given a revenue function and cost function

4.8 L'Hospital's Rule

- L'Hospital's Rule with an Indeterminate Quotient
 - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a quotient of the form $(0/0)$
 - Apply L'Hospital's rule in the $0/0$ case
 - Apply L'Hospital's rule in the $0/0$ case with a sine or cosine function
 - Apply L'Hospital's rule in the $0/0$ case with another trigonometric function
 - Apply L'Hospital's rule in the $0/0$ case with an exponential function
 - Apply L'Hospital's rule in the $0/0$ case with a logarithmic function
 - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a quotient of the form (∞/∞)
 - Apply L'Hospital's rule in the (∞/∞) case
 - Apply L'Hospital's rule in the (∞/∞) case with an exponential function
 - Apply L'Hospital's rule in the (∞/∞) case with a logarithmic function
 - L'Hospital's Rule with an Indeterminate Product, Difference, or Power
 - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a product
 - Apply L'Hospital's rule in the $0 \cdot \infty$ case with an exponential function
 - Apply L'Hospital's rule in the $0 \cdot \infty$ case with a logarithmic function
-

- Apply L'Hospital's rule in the $0 \cdot \infty$ case with a sine or cosine function
- Apply L'Hospital's rule in the $0 \cdot \infty$ case with another trigonometric function
- Apply L'Hospital's rule to find the limit of an indeterminate form produced by a difference
 - Apply L'Hospital's rule in the infinity - infinity case
 - Apply L'Hospital's rule in the infinity - infinity case with a sine or cosine function
 - Apply L'Hospital's rule in the infinity - infinity case with an exponential function
 - Apply L'Hospital's rule in the infinity - infinity case with a logarithmic function
- Apply L'Hospital's rule to find the limit of an indeterminate form produced by a power
 - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a power
 - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a power involving a sine or cosine function
 - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a power involving a logarithmic function
 - Apply L'Hospital's rule to find the limit of an indeterminate form produced by a power involving an exponential function
- Determine when a limit is an indeterminate form
- Growth Rates of Functions Using L'Hospital's Rule
 - Use L'Hospital's rule to compare the growth rate of two functions
 - Use L'Hospital's rule to compare the growth rate of two functions
 - Use L'Hospital's rule to compare the growth rate of two functions, one of which is an exponential function with base e
 - Use L'Hospital's rule to compare the growth rate of two functions, one of which is an exponential function with base other than e
 - Use L'Hospital's rule to compare the growth rate of two functions, one of which is a logarithmic function

4.9 Newton's Method

- Newton's Method
 - Set up an equation to find a value using Newton's method
 - Set up an equation to use Newton's method to find the root of a polynomial function
 - Set up an equation to use Newton's method to solve a trigonometric equation
 - Set up an equation to use Newton's method to solve an exponential equation
 - Set up an equation to use Newton's method to solve a logarithmic equation
 - Use Newton's method to approximate the root of a polynomial function or the solution of an algebraic equation
 - Find the root of a polynomial function using Newton's method
 - Solve a polynomial equation using Newton's method
-

- Solve a rational equation using Newton's method
- Solve a radical equation using Newton's method
- Use Newton's method to approximate the value of a radical expression
 - Set up an equation to approximate the value of a radical expression using Newton's method
 - Use Newton's method to approximate the value of a radical expression
- Use Newton's method to approximate the solution of a trigonometric or exponential equation
 - Solve an equation involving sine or cosine using Newton's method
 - Solve an equation involving an exponential function using Newton's method
 - Solve an equation involving a logarithmic function using Newton's method
- Determine when Newton's method does not work
- Use Newton's method in application
 - Use Newton's method to find the location of local extrema for polynomial functions
 - Use Newton's method with Kepler's equation regarding planetary orbits

4.10 Antiderivatives

- The Antiderivative
 - Find the general antiderivative of an algebraic function
 - Find the general antiderivative of a polynomial function
 - Find the general antiderivative of an algebraic function containing negative exponents
 - Find the general antiderivative of an algebraic function containing rational exponents
 - Find the general antiderivative of a trigonometric function
 - Find the general antiderivative of a sine or cosine function
 - Find the general antiderivative of other trigonometric functions
 - Find the general antiderivative of an exponential or logarithmic function
 - Find the general antiderivative of an exponential function
 - Find the general antiderivative of a logarithmic function
 - Find the general antiderivative of an inverse trigonometric function
 - The Indefinite Integral
 - Understand integral notation and verify an indefinite integral
 - Verify an indefinite integral involving a polynomial
 - Verify an indefinite integral involving a trigonometric expression
 - Verify an indefinite integral involving an exponential expression
 - Understand the properties of indefinite integrals
 - Evaluate indefinite integrals involving algebraic functions
 - Evaluate an indefinite integral of a polynomial expression
-

- Evaluate an indefinite integral of an algebraic expression containing negative exponents
- Evaluate an indefinite integral of an algebraic expression containing rational exponents
- Evaluate indefinite integrals involving trigonometric functions
 - Evaluate an indefinite integral of a sine or cosine function
 - Evaluate an indefinite integral of other trigonometric function
- Evaluate indefinite integrals involving exponential or logarithmic function
 - Evaluate an indefinite integral of an exponential function
 - Evaluate an indefinite integral of a logarithmic function
- Evaluating indefinite integrals involving inverse trigonometric functions
- Initial-Value Problems and Motion
 - Solve an initial-value problem
 - Solve an initial value problem for a polynomial function
 - Solve an initial value problem with roots or rational expressions
 - Solve an initial value problem for a trigonometric function involving sine or cosine
 - Solve an initial value problem for trigonometric functions other than sine or cosine
 - Solve an initial value problem given a second derivative and initial values for the first derivative and the function
 - Solve an initial value problem given a second derivative and two initial values for the function
 - Solve a word problem involving antiderivatives and motion
 - Find a position function given a velocity function and initial position
 - Find a position function given an acceleration function and an initial velocity and an initial position
 - Solve a word problem by finding a position function given initial conditions
 - Find the maximum height of a projectile
 - Find the time a projectile is in the air

Chapter 5: Integration

5.1 Approximating Areas

- Sigma Notation
 - Write a series of numbers using sigma notation and use the properties of sigma notation
 - Write a series of numbers using sigma notation
 - Evaluate a sum given in sigma notation
 - Use the properties of sigma notation
 - Use sigma notation to calculate sums and powers of integers
 - Approximating Area
-

- Approximate the area under a curve using rectangles
 - Use rectangles to graphically represent a left-endpoint approximation of the area under a curve
 - Use rectangles to graphically represent a right-endpoint approximation of the area under a curve
 - Approximate the area under a curve using a left-endpoint approximation given a graph
 - Approximate the area under a curve using a right-endpoint approximation given a graph
 - Approximate the area under a curve using a left-endpoint approximation given a function
 - Approximate the area under a curve using a right-endpoint approximation given a function
- Approximate the area under a curve using midpoint approximation
 - Use rectangles to graphically represent a midpoint approximation of the area under a curve
 - Approximate the area under a curve using a midpoint approximation given a graph
 - Approximate the area under a curve using a midpoint approximation given a function
- Find an upper or lower sum given a function
 - Find the lower sum given a function
 - Find the upper sum given a function
 - Graph a visualization of an upper or lower sum
- Determine if a sum will be an underestimate or overestimate
- Solve application problems involving approximating areas

5.2 The Definite Integral

- Definition and Notation of Definite Integrals
 - Identify notation of a definite integral and describe when a function is integrable
 - Express a limit or a Riemann sum as a definite integral
 - Express a limit as a definite integral
 - Express a Riemann sum as a definite integral
 - Evaluate an integral using the definition of the definite integral and left- or right-endpoint approximations
 - Express a definite integral using a Riemann sum
 - Evaluate a definite integral using a Riemann sum
 - Area and the Definite Integral
 - Use a geometric formula to calculate a definite integral
-

- Use a geometric formula to calculate a definite integral of a linear function given a graph
 - Use a geometric formula to calculate a definite integral of an absolute value function given a graph
 - Use a geometric formula to calculate a definite integral of a radical function given a graph
 - Use a geometric formula to calculate a definite integral of a piecewise function given a graph
 - Calculate net signed areas under a function using geometric formulas
 - Use a geometric formula to find the net signed area of a linear function given a graph
 - Use a geometric formula to find the net signed area of an absolute value function given a graph
 - Use a geometric formula to find the net signed area of a piecewise function given a graph
 - Use a geometric formula to find the net signed area of a linear function
 - Use a geometric formula to find the net signed area of an absolute value function
 - Calculate total area under a function using geometric formulas
 - Use a geometric formula to find the total area of a linear function given a graph
 - Use a geometric formula to find the total area of an absolute value function given a graph
 - Use a geometric formula to find the total area of a piecewise function given a graph
 - Use a geometric formula to find the total area of a linear function
 - Use a geometric formula to find the total area of an absolute value function
 - Properties of the Definite Integral and the Average Value of a Function Geometrically
 - Use the properties of the definite integral
 - Understand the comparison theorem of integrals and use it to compare two functions over a given interval
 - Calculate the average value of a function using a geometric argument
 - Find the average value of a function given the value of an integral
 - Calculate the average value of a function given a graph
 - Calculate the average value of a linear function using a geometric argument
 - Calculate the average value of a radical function using a geometric argument
 - Use the Mean Value Theorem for Integrals and a geometric argument to find where a function takes on its mean value
 - Find the point on a function where the function takes on its mean value
 - Use the Mean Value Theorem for Integrals to find where a linear function takes on its average value
-

5.3 The Fundamental Theorem of Calculus

- Fundamental Theorem of Calculus Part 1 - Integrals and Antiderivatives
 - Use the Fundamental Theorem of Calculus to find the derivative of an integral function
 - Use the Fundamental Theorem of Calculus to find the derivative of an integral function
 - Use the Fundamental Theorem of Calculus and a property of integrals to find the derivative of an integral function
 - Use the Fundamental Theorem of Calculus to find the derivative of an integral trigonometric function
 - Use the Fundamental Theorem of Calculus to find the derivative of an integral exponential or logarithmic function
 - Use the Fundamental Theorem of Calculus and the chain rule to find a derivative of an integral function
 - Use the Fundamental Theorem of Calculus and the chain rule to find a derivative of an integral function
 - Use the Fundamental Theorem of Calculus and the chain rule to find a derivative of an integral function with a radical bound
 - Use the Fundamental Theorem of Calculus and the chain rule to find a derivative of an integral function with a trigonometric bound
 - Use the Fundamental Theorem of Calculus and the chain rule to find a derivative of an integral function with an exponential or logarithmic bound
 - Use the Fundamental Theorem of Calculus with two variable limits of integration
 - Use the Fundamental Theorem of Calculus with two variable limits of integration
 - Use the Fundamental Theorem of Calculus with two variable limits of integration and a radical bound
 - Use the Fundamental Theorem of Calculus with two variable limits of integration and a trigonometric bound
 - Use the Fundamental Theorem of Calculus with two variable limits of integration and an exponential or logarithmic bound
 - Fundamental Theorem of Calculus Part 2 - The Evaluation Theorem
 - Understand the Fundamental Theorem of Calculus graphically
 - Use the Fundamental Theorem of Calculus to relate a function defined by an integral and area under a curve
 - Use the Fundamental Theorem of Calculus to relate a function defined by an integral and its derivative
 - Use the Fundamental Theorem of Calculus to relate a function defined by an integral and its behavior
 - Evaluate definite integrals involving algebraic functions using the Fundamental Theorem of Calculus
-

- Evaluate a definite integral of a polynomial expression
- Evaluate a definite integral of an algebraic expression containing negative exponents
- Evaluate a definite integral of an algebraic expression containing rational exponents
- Evaluate definite integrals involving trigonometric functions
 - Evaluate a definite integral of a sine or cosine function
 - Evaluate a definite integral of other trigonometric functions
- Average Value of a Function with the Fundamental Theorem of Calculus
 - Calculate the average value of a function
 - Calculate the average value of a polynomial function
 - Calculate the average value of an algebraic expression containing negative exponents
 - Calculate the average value of an algebraic expression containing rational exponents
 - Calculate the average value of a sine or cosine function
 - Calculate the average value of other trigonometric functions
 - Use the Mean Value Theorem for Integrals to find the point on the curve which takes on the average value of the function

5.4 Net Change Theorem

- Introduction to the Net Change Theorem and Displacement Problems
 - Use the net change theorem to solve problems given graphs
 - Use the net change theorem given a graph of a function
 - Use the net change theorem given a graph of a derivative function
 - Find the net displacement of a particle given its velocity function
 - Find the net displacement of a particle given its velocity function
 - Find the net displacement of a particle given a velocity function with negative exponents
 - Find the net displacement of a particle given a velocity function with rational exponents
 - Find the net displacement of a particle given a velocity function with a sine or cosine function
 - Find the total distance traveled by a particle given its velocity function
 - The Net Change Theorem in Other Contexts
 - Use the net change theorem to find amounts given rates of change in a scientific context
 - Use the net change theorem to find a change in volume
 - Use the net change theorem to find a population change
 - Use the net change theorem to find the net charge through a wire
-

- Use the net change theorem to find the mass of a rod given its linear density
- Use the net change theorem to find amounts given rates of change in a business or social science context
 - Use the net change theorem to find a change in cost given the marginal cost function
 - Use the net change theorem to find a change in revenue given the marginal revenue function
- Use the net change theorem to find amounts given rates of change in a geometric context
 - Use the net change theorem to find amounts given rates of change in a geometric context
- Integrating Even and Odd Functions
 - Integrate an even or odd function
 - Find a definite integral using the properties of even functions
 - Find a definite integral using the properties of odd functions

5.5 Substitution

- U-Substitution and Indefinite Integrals
 - Use substitution to find an indefinite integral with the power rule and u given
 - Use substitution to find an indefinite integral with the power rule and u given
 - Use substitution to find an indefinite integral with the power rule and u given but with further manipulation needed
 - Use substitution to find an indefinite integral with the power rule for negative exponents and u given
 - Use substitution to find an indefinite integral with the power rule for negative exponents and u given but with further manipulation needed
 - Use substitution to find an indefinite integral with the power rule for rational exponents and u given
 - Use substitution to find an indefinite integral with the power rule for rational exponents and u given but with further manipulation needed
 - Use substitution to evaluate an indefinite integral with the power rule
 - Use substitution to evaluate an indefinite integral with the power rule
 - Use substitution to evaluate an indefinite integral with the power rule where further manipulation may be necessary
 - Use substitution to find an indefinite integral with the power rule for negative exponents
 - Use substitution to find an indefinite integral with the power rule for rational exponents
 - Use substitution to find an indefinite integral with trigonometric functions
 - Use substitution to find an indefinite integral with sine or cosine functions
-

- Use substitution to find an indefinite integral with other trigonometric functions
- U-Substitution and Definite Integrals
 - Use substitution to evaluate a definite integral with the power rule
 - Determine the new limits of integration when evaluating a definite integral using substitution
 - Use substitution to evaluate a definite integral with the power rule
 - Use substitution to evaluate a definite integral with the power rule and negative exponents
 - Use substitution to evaluate a definite integral with the power rule and rational exponents
 - Use substitution to evaluate a definite integral with trigonometric functions
 - Determine the new limits of integration when evaluating a definite integral using substitution for trigonometric functions
 - Use substitution to evaluate a definite integral with sine or cosine functions
 - Use substitution to evaluate a definite integral with other trigonometric functions

5.6 Integrals Involving Exponential and Logarithmic Functions

- Indefinite Integrals with Exponential and Logarithmic Functions
 - Use substitution to find an indefinite integral with exponential functions
 - Use substitution to find an indefinite integral with exponential functions of base e
 - Use substitution to find an indefinite integral with exponential functions of a base other than e
 - Use substitution to find an indefinite integral with exponential functions of base e and negative exponents
 - Use substitution to find an indefinite integral with exponential functions of base e and rational exponents
 - Use substitution to find an indefinite integral involving exponential functions and trigonometric functions
 - Use substitution to find an indefinite integral involving exponential functions and sine or cosine functions
 - Use substitution to find an indefinite integral involving exponential functions and other trigonometric functions
 - Use substitution to find an indefinite integral involving logarithmic functions
 - Use substitution to find an indefinite integral of a rational function
 - Use substitution to find an indefinite integral involving natural logarithmic functions
 - Find an indefinite integral of a natural logarithmic function
 - Find an indefinite integral of a logarithmic function of another base
 - Use substitution to find an indefinite integral involving logarithmic functions and trigonometric functions
-

- Use substitution to find an indefinite integral of a tangent function
- Use substitution to find an indefinite integral involving natural logarithm and trigonometric functions
- Definite Integrals with Exponential and Logarithmic Functions and Applications
 - Use substitution to evaluate a definite integral with exponential functions
 - Use substitution to evaluate a definite integral with rational or logarithmic functions
 - Use substitution to evaluate a definite integral involving logarithmic functions
 - Use substitution to evaluate a definite integral involving logarithmic functions and trigonometric functions
 - Solve application problems using integrals of exponential functions
 - Solve an application involving population with base e
 - Find a price-demand equation given the marginal price-demand function with base e
 - Solve an application involving population with base other than e
 - Solve application problems using integrals involving logarithmic functions
 - Solve an application problem involving a rational function
 - Solve an application problem involving a natural logarithmic function
 - Solve an application problem involving a logarithmic function of another base

5.7 Integrals Resulting in Inverse Trigonometric Functions

- Integrals with Inverse Trigonometric Functions
 - Use substitution to find an indefinite integral involving an inverse trigonometric function
 - Use substitution to find an indefinite integral resulting in an inverse trigonometric function
 - Use substitution to find an indefinite integral involving an inverse trigonometric function
 - Use substitution to evaluate a definite integral involving an inverse trigonometric function

Chapter 6: Applications of Integration

6.1 Areas Between Curves

- Area Between Curves
 - Determine the area of a region between two curves defined by algebraic functions
 - Represent the area between two curves graphically
 - Write an integral to represent the area between two curves
 - Find the area of a region between two linear functions
 - Find the area of a region between two functions with a polynomial
 - Find the area of a region between two functions with rational exponents
 - Find the area of a region between two functions with negative exponents
 - Determine the area of a region between two curves involving transcendental functions

- Find the area of a region between two curves with a sine or cosine function
- Find the area of a region between two curves with an exponential function
- Find the area of a region between two curves where integrating results in a natural logarithm
- Area of Compound Regions
 - Determine the area of a compound region defined by algebraic functions
 - Find the area bounded by two functions that intersect
 - Find the area bounded above or below by two different functions
 - Find the area bounded above or below by a radical function and another function
 - Find the area bounded above or below by a rational function and another function
 - Determine the area of a compound region involving transcendental functions
 - Find the area bounded by two trigonometric functions that cross
 - Find the area bounded above or below by an exponential function and another function
 - Find the area bounded above or below by functions where integrating results in a natural logarithmic function
- Area Between Curves for Functions of y and Applications
 - Determine the area of a region between two curves defined by algebraic functions by integrating with respect to the dependent variable
 - Write an integral to represent the area between two curves defined as functions of y
 - Find the area between two linear functions defined as functions of y
 - Find the area between two functions defined as functions of y
 - Find the area between two functions defined as functions of y with rational exponents
 - Determine the area of a region between two curves by rewriting functions in terms of y
 - Rewrite functions as functions of y to make a compound region a single integral
 - Find the area between two curves by rewriting functions as functions of y
 - Find the area between two curves by rewriting functions as functions of y with rational exponents
 - Use the area between curves to solve application problems
 - Find the total profit given marginal cost and marginal revenue functions
 - Find the distance between objects given two velocity functions
 - Find a total accumulation given two rate of change functions

6.2 Determining Volumes by Slicing

- Determine Volumes Using Cross-Sections
 - Determine the volume of a solid by integrating a cross-section
-

- Find the volume of a solid with a base formed by two functions and square cross sections
 - Find the volume of a solid with a base formed by two functions and equilateral triangle cross sections
 - Find the volume of a solid with a base formed by two functions and semicircle cross sections
 - Find the volume of a solid with cross sections and a circular base
 - Determine Volumes with the Disc Method
 - Find the volume of a solid of revolution using the disk method
 - Set up an integral to find the volume of a solid of revolution around the x-axis with the disc method
 - Use the disc method to find the volume of a solid of revolution with a polynomial function
 - Use the disc method to find the volume of a solid of revolution with a function with rational exponents
 - Use the disc method to find the volume of a solid of revolution with a sine or cosine function
 - Use the disc method to find the volume of a solid of revolution with an exponential function
 - Set up an integral to find the volume of a solid of revolution around the y-axis with the disc method
 - Determine Volumes with the Washer Method
 - Find the volume of a solid of revolution with a cavity using the washer method
 - Set up an integral to find the volume of a solid of revolution using the washer method
 - Use the washer method to find the volume of a solid of revolution with a polynomial function
 - Use the washer method to find the volume of a solid of revolution with a function with rational exponents
 - Use the washer method to find the volume of a solid of revolution with an exponential function
 - Set up an integral to find the volume of a solid of revolution around a line other than the x-axis using the washer method
 - Find the volume of a solid of revolution using the washer method for functions of y
 - Set up an integral to find the volume of a solid of revolution using the washer method for a function of y
 - Use the washer method to find the volume of a solid of revolution with a polynomial function in terms of y
-

- Use the washer method to find the volume of a solid of revolution with a function in terms of y with rational exponents
- Set up an integral to find the volume of a solid of revolution around a line other than the y -axis using the washer method for functions of y
- Find the volume of a solid of revolution using the washer method by rewriting a function as a function of y
 - Set up an integral to find the volume of a solid of revolution using the washer method by rewriting a function as a function of y
 - Use the washer method to find the volume of a solid of revolution by rewriting a function in terms of y
 - Use the washer method to find the volume of a solid of revolution by rewriting a function in terms of y with rational exponents
 - Set up an integral to find the volume of a solid of revolution around a line other than the y -axis using the washer method and rewriting functions in terms of y

6.3 Volumes of Revolution - Cylindrical Shells

- Determine Volumes with the Shell Method
 - Calculate the volume of a solid of revolution by using the method of cylindrical shells
 - Set up an integral to find the volume of a solid of revolution using the shell method
 - Use the shell method to find the volume of a solid of revolution with a polynomial function
 - Use the shell method to find the volume of a solid of revolution with a function with rational exponents
 - Use the shell method to find the volume of a solid of revolution with a sine or cosine function
 - Use the shell method to find the volume of a solid of revolution with an exponential function
 - Set up an integral to find the volume of a solid of revolution around a line other than the y -axis using the shell method
 - Calculate the volume of a solid of revolution by using the method of cylindrical shells for functions of y
 - Set up an integral to find the volume of a solid of revolution using the shell method for a function of y
 - Use the shell method to find the volume of a solid of revolution with a polynomial function in terms of y
 - Use the shell method to find the volume of a solid of revolution with a function in terms of y with rational exponents
 - Set up an integral to find the volume of a solid of revolution around a line other than the x -axis using the shell method for functions of y
-

- Compare Methods for Volumes of Revolution
 - Choose an appropriate method to find the volume of a solid of revolution

6.4 Arc Length of a Curve and Surface Area

- Calculating Arc Length
 - Determine the length of a curve $f(x)$ between two points
 - Set up an integral to find an arc length
 - Find the arc length of a linear function
 - Find the arc length of a function with rational exponents
 - Find the arc length of a function with negative exponents
 - Use a calculator to find the arc length of a function
 - Determine the length of a curve $g(y)$ between two points
 - Set up an integral to find the arc length of a curve of $g(y)$
 - Find the arc length of a linear function given as $g(y)$
 - Find the arc length of a function with negative exponents given as $g(y)$
 - Use a calculator to find the arc length of a function given as $g(y)$
 - Rewrite a function given as $f(x)$ to $g(y)$ to find arc length
- Area of a Surface of Revolution
 - Find the surface area of a surface of revolution around the x-axis
 - Set up an integral to find the surface area from revolving a curve around the x-axis
 - Find the surface area of a revolution of a polynomial function around the x-axis
 - Find the surface area of a revolution of a function with rational exponents around the x-axis
 - Use a calculator to find the surface area of a surface of revolution around the x-axis
 - Find the surface area of a surface of revolution with a function $g(y)$
 - Set up an integral to find the surface area from revolving a curve around the y-axis
 - Find the surface area of a revolution of a polynomial function around the y-axis
 - Use a calculator to find the surface area of a surface of revolution around the y-axis
 - Rewrite a function given as $f(x)$ to $g(y)$ to find surface area

6.5 Physical Applications

- Mass and Density
 - Determine the mass of a one-dimensional object from its linear density function
 - Determine the mass of a one-dimensional object given a polynomial linear density function
 - Determine the mass of a one-dimensional object given a linear density function with rational exponents
-

- Determine the mass of a one-dimensional object given a linear density function with negative exponents
 - Determine the mass of a one-dimensional object from a sine or cosine linear density function
 - Determine the mass of a one-dimensional object from an exponential linear density function
 - Determine the mass of a two-dimensional object from its radial density function
 - Determine the mass of a two-dimensional object given a polynomial radial density function
 - Determine the mass of a two-dimensional object given a radial density function with rational exponents
 - Determine the mass of a two-dimensional object given a radial density function with negative exponents
 - Determine the mass of a two-dimensional object given a sine or cosine radial density function
 - Determine the mass of a two-dimensional object given an exponential radial density function
 - Work Done by a Force
 - Calculate the work done by a force acting along a line
 - Find the amount of work done given a constant force
 - Find the amount of work done to lift an object given its mass
 - Calculate the work done by a variable force acting along a line
 - Calculate the work done to compress or stretch a spring
 - Calculate the amount of work done to compress or stretch a spring given an initial force
 - Calculate the amount of work done to compress or stretch a spring given an initial work
 - Find the length of a spring given two initial amounts of work
 - Calculate the work done to lift a cable
 - Calculate the amount of work necessary to lift a cable
 - Calculate the amount of work necessary to lift a cable and an attached object
 - Calculate the amount of work necessary to lift a cable and an attached object that is losing mass
 - Work Done in Pumping and Hydrostatic Force
 - Calculate the work done in pumping a liquid out of a tank
 - Calculate the work done in pumping a liquid out of a cylindrical tank
 - Calculate the work done in pumping a liquid out of a tank in the shape of an inverted cone
 - Find the hydrostatic force against a submerged plate
-

- Calculate the hydrostatic force on a submerged horizontal plate
- Calculate the hydrostatic force on a submerged vertical plate in the shape of an isosceles triangle
- Calculate the hydrostatic force on a submerged vertical plate in the shape of a trapezoid
- Calculate the hydrostatic force on a submerged vertical plate in the shape of a circle or semicircle

6.6 Moments and Centers of Mass

- Centers of Mass on a Line or Plane
 - Find the moment and center of mass of objects distributed along a line
 - Find the moment of objects distributed along a line
 - Find the center of mass of objects distributed along a line
 - Represent the center of mass of objects distributed along a line graphically
 - Find the moments and the center of mass of objects in a plane
 - Find the moments of objects in a plane
 - Find the center of mass of objects in a plane
 - Represent the center of mass of objects distributed in a plane graphically
- Centroids of Regions on a Plane
 - Find the mass, moments, and center of mass of a lamina with uniform density
 - Find the mass of a lamina
 - Find the moments of a lamina
 - Find the center of mass of a lamina bounded by a function using the mass and moments
 - Find the center of mass of a lamina bounded by two functions using the mass and moments
 - Find the centroid of a lamina of uniform density using the area
 - Find the centroid of a lamina of uniform density bounded by a function using the area
 - Find the centroid of a lamina bounded by two functions using the area
 - Use symmetry to help locate the centroid of a thin plate
 - Apply the theorem of Pappus for volume
 - Use the Theorem of Pappus to find the volume of a solid given coordinate points
 - Use the Theorem of Pappus to find the volume of a torus

6.7 Exponential Growth and Decay

- Exponential Growth Models
 - Use an exponential growth model to solve problems involving population
 - Find a population given a function
 - Find when a population will reach a given number given a function
 - Write a function to model population growth
-

- Solve compound interest problems
 - Solve compound interest problems with discrete compounding intervals
 - Use continuously compounding interest to find a new balance
 - Use continuously compounding interest to find when an account will reach a given balance
 - Use continuously compounding interest to find the principal required to have a given balance
- Use the concept of doubling time to solve exponential growth problems
 - Find the amount of time necessary for an initial value to reach a given multiple
 - Use the concept of doubling time to find a growth rate
 - Use the concept of doubling time to solve exponential growth problems
- Exponential Decay Models
 - Use an exponential decay model to solve problems involving Newton's Law of Cooling
 - Write a function to model temperature following Newton's Law of Cooling
 - Use Newton's Law of Cooling to find the temperature of an object
 - Use Newton's Law of Cooling to find when an object will reach a given temperature
 - Use the concept of half-life to solve radioactive decay problems
 - Write a function to model exponential decay in half-life problems
 - Determine the mass of a sample remaining in a half-life problem
 - Determine the age of a sample given a resultant mass in a half-life problem

6.8 Calculus of the Hyperbolic Functions

- Derivatives and Integrals of the Hyperbolic Functions
 - Find the derivative of a hyperbolic function
 - Find the derivative of a hyperbolic sine or cosine function
 - Find the derivative of a hyperbolic tangent or cotangent function
 - Find the derivative of a hyperbolic secant or cosecant function
 - Evaluate an integral involving a hyperbolic function
 - Evaluate an indefinite integral involving a hyperbolic sine or cosine function
 - Evaluate an indefinite integral involving a hyperbolic tangent or cotangent function
 - Evaluate an indefinite integral involving a hyperbolic secant or cosecant function
 - Use a catenary to find the length of a hanging cable
 - Calculus of Inverse Hyperbolic Functions
 - Find the derivative of an inverse hyperbolic function
 - Find the derivative of an inverse hyperbolic sine or cosine function
 - Find the derivative of an inverse hyperbolic tangent or cotangent function
 - Find the derivative of an inverse hyperbolic secant or cosecant function
 - Evaluate an integral involving an inverse hyperbolic function
-

- Evaluate an indefinite integral involving an inverse hyperbolic sine or cosine function
- Evaluate an indefinite integral involving an inverse hyperbolic tangent or cotangent function
- Evaluate an indefinite integral involving an inverse hyperbolic secant or cosecant function

Chapter 7: Techniques of Integration

7.1 Integration by Parts

- Integration by Parts for Indefinite Integrals
 - Use integration by parts to find indefinite integrals given u and dv
 - Use the integration by parts formula to solve an indefinite integral with a logarithmic function, given u and dv
 - Use the integration by parts formula to solve an indefinite integral with an exponential function given u and dv
 - Use the integration by parts formula to solve an indefinite integral with a trigonometric function, given u and dv
 - Use integration by parts to find indefinite integrals
 - Use the integration by parts formula once to solve an indefinite integral with a logarithmic function
 - Use the integration by parts formula once to solve an indefinite integral with an exponential function
 - Use the integration by parts formula once to solve an indefinite integral with a trigonometric function
 - Use the integration by parts formula to solve an indefinite integral with a single function
 - Use integration by parts more than once to find indefinite integrals
 - Use the integration by parts formula more than once to solve an indefinite integral with a logarithmic function
 - Use the integration by parts formula more than once to solve an indefinite integral with an exponential function
 - Use the integration by parts formula more than once to solve an indefinite integral with a trigonometric function
 - Use the integration by parts formula to identify a pattern that solves an indefinite integral with a logarithmic function
 - Use the integration by parts formula to identify a pattern that solves an indefinite integral with an exponential function
 - Integration by Parts for Definite Integrals and Applications
 - Use integration by parts for definite integrals
-

- Use the integration by parts formula to evaluate definite integrals with logarithmic functions
- Use the integration by parts formula to evaluate definite integrals with exponential functions
- Use the integration by parts formula to evaluate definite integrals with trigonometric functions
- Use integration by parts to solve area and volume problems
 - Use the integration by parts formula to find the area of a region with logarithmic or exponential functions
 - Use the integration by parts formula to find the area of a region with trigonometric functions
 - Use the integration by parts formula to find the volume of a solid with logarithmic or exponential functions
 - Use the integration by parts formula to find the volume of a solid with trigonometric functions

7.2 Trigonometric Integrals

- Integrating Products and Powers of Trigonometric Functions
 - Solve trigonometric integrals with products and powers of sine and cosine
 - Solve trigonometric integrals with products and powers of sine and cosine where all exponents are odd
 - Solve trigonometric integrals with products and powers of sine and cosine where one exponent is odd
 - Solve trigonometric integrals with products and powers of sine and cosine where all exponents are even
 - Solve definite integrals with products and powers of sine and cosine where all exponents are odd
 - Solve definite integrals with products and powers of sine and cosine where one exponent is odd
 - Solve definite integrals with products and powers of sine and cosine where all exponents are even
 - Solve trigonometric integrals with products and powers of tangent and secant
 - Solve trigonometric integrals with products and powers of tangent and secant where the secant function has an even power
 - Solve trigonometric integrals with products and powers of tangent and secant where all exponents are odd
 - Solve trigonometric integrals with products and powers of tangent
 - Solve trigonometric integrals with products and powers of secant using a reduction formula
 - Solve definite integrals with products and powers of tangent and secant
-

- Reduction Formulas
 - Solve trigonometric integrals with different angles using product-to-sum identities
 - Solve trigonometric integrals of one sine and one cosine function with different angles using product to sum identities
 - Solve trigonometric integrals of two sine functions with different angles using product to sum identities
 - Solve trigonometric integrals of two cosine functions with different angles using product to sum identities

7.3 Trigonometric Substitution

- Integration by Trigonometric Substitution
 - Solve integration problems involving a difference of squares of the form a^2-x^2 using a trigonometric substitution
 - Evaluate an indefinite integral containing a difference of squares of the form a^2-x^2
 - Evaluate a definite integral containing a difference of squares of the form a^2-x^2
 - Solve integration problems involving a sum of squares using a trigonometric substitution
 - Evaluate an indefinite integral containing a sum of squares
 - Evaluate a definite integral containing a sum of squares
 - Use a hyperbolic trigonometric substitution to evaluate an indefinite integral containing a sum of squares
 - Solve integration problems involving a difference of squares of the form x^2-a^2 using a trigonometric substitution
 - Evaluate an indefinite integral containing a difference of squares of the form x^2-a^2
 - Evaluate a definite integral containing a difference of squares of the form x^2-a^2
 - Use a hyperbolic trigonometric substitution to evaluate an indefinite integral containing a difference of squares of the form x^2-a^2
 - Applications and Extensions of Integration by Trigonometric Substitution
 - Solve application problems involving trigonometric substitution
 - Use a trigonometric substitution to find an arc length
 - Find the area of a region under a curve using integration involving a trigonometric substitution
 - Solve integration problems requiring the process of completing the square
 - Use the techniques of completing the square and a sine substitution to evaluate an integral
 - Use the techniques of completing the square and a tangent substitution to evaluate an integral
-

- Use the techniques of completing the square and a secant substitution to evaluate an integral

7.4 Partial Fractions

- Integration Using Polynomial Long Division
 - Integrate a rational function using long division of polynomials
- Integration Using Partial Fractions
 - Integrate a rational function using partial fractions with nonrepeated linear factors
 - Integrate a rational function using partial fractions with nonrepeated linear factors
 - Integrate a rational function using long division and partial fractions with nonrepeated linear factors
 - Integrate a rational function using partial fractions with nonrepeated linear factors after a substitution
 - Integrate a rational function using partial fractions with repeated linear factors or a quadratic factor
 - Integrate a rational function using partial fractions with repeated linear factors
 - Integrate a rational function using partial fractions with a nonrepeated quadratic factor
 - Integrate a rational function using partial fractions with a repeated quadratic factor
 - Integrate a function using partial fractions in applications

7.5 Other Strategies for Integration

- Integration Using a Table of Integrals
 - Use a table of integrals to evaluate integral problems
 - Determine the appropriate formula to evaluate the integral
 - Evaluate an integral given an integral formula
 - Evaluate an integral given a table of integrals
 - Evaluate an integral given a table of integrals with u-substitution
 - Integration Using a Computer Algebra System
 - Use a computer algebra system (CAS) to solve integral problems
 - Use a CAS to evaluate integrals with polynomials or rational functions
 - Use a CAS to evaluate integrals with trigonometric functions
 - Use Desmos to evaluate definite integrals
 - Use a computer algebra system (CAS) to solve application problems with integrals
 - Solve area application problems with a CAS
 - Solve area application problems with Desmos
 - Solve volume application problems with a CAS
 - Solve volume application problems with Desmos
-

7.6 Numerical Integration

- Trapezoidal Rule
 - Approximate a definite integral using the Trapezoid Rule
 - Approximate the area under a curve using the Trapezoid method given a graph
 - Approximate the area under a curve using the Trapezoid method given a function
 - Represent a trapezoid approximation of the area under a curve graphically
- Absolute and Relative Error
 - Determine the absolute and relative error in using the midpoint or trapezoidal rules
 - Calculate the absolute error in using a midpoint approximation
 - Calculate the absolute error in using a trapezoidal approximation
 - Calculate the relative error in using a midpoint approximation
 - Calculate the relative error in using a trapezoidal approximation
 - Determine error bounds for midpoint and trapezoidal rules
 - Calculate an upper bound for the error using the midpoint rule
 - Calculate an upper bound for the error using the Trapezoid rule
 - Determine the number of intervals necessary to reach a given level of accuracy using the midpoint rule
 - Determine the number of intervals necessary to reach a given level of accuracy using the trapezoid rule
- Simpson's Rule
 - Approximate a definite integral using Simpson's rule
 - Approximate the area under a curve using Simpson's rule given a graph
 - Approximate the area under a curve using Simpson's rule given a table
 - Approximate the area under a curve using Simpson's rule given a function
 - Determine error and error bounds in using Simpson's rule
 - Calculate the absolute error in using Simpson's rule
 - Calculate the relative error in using Simpson's rule
 - Calculate an upper bound for the error using Simpson's rule
 - Determine the number of intervals necessary to reach a given level of accuracy using Simpson's rule

7.7 Improper Integrals

- Integrating over an Infinite Interval
 - Evaluate an improper integral over an infinite interval
 - Evaluate improper integrals that are continuous over an infinite interval
 - Evaluate improper integrals that are continuous over an infinite interval using L'Hospital's Rule
 - Integrating a Discontinuous Integrand
 - Evaluate an improper integral with an infinite discontinuity
-

- Evaluate an improper integral with an infinite discontinuity at one endpoint
- Evaluate an improper integral of a trigonometric function with an infinite discontinuity at one endpoint
- Evaluate an improper integral with an infinite discontinuity within the limits of integration
- Evaluate an improper integral with an infinite discontinuity at one endpoint using L'Hospital's Rule
- The Comparison Theorem and Convergence
 - Use the Comparison Theorem to determine whether an improper integral is convergent
 - Determine whether an improper integral with rational functions is convergent using the Comparison Theorem
 - Demonstrate a conceptual understanding of the Comparison Theorem
 - Determine whether an improper integral with exponential functions is convergent using the Comparison Theorem
 - Determine whether an improper integral with logarithmic functions is convergent using the Comparison Theorem
 - Determine whether an improper integral with trigonometric functions is convergent using the Comparison Theorem

Chapter 8: Introduction to Differential Equations

8.1 Basics of Differential Equations

- General Solutions to Differential Equations
 - Identify basic elements of differential equations
 - Identify the order of a differential equation
 - Verify that a polynomial or rational function is a general solution of a differential equation
 - Verify that a trigonometric function is a general solution of a differential equation
 - Verify that an exponential function is a general solution of a differential equation
 - Verify that a logarithmic function is a general solution of a differential equation
 - Solve general solution problems for linear differential equations
 - Find a general solution to a differential equation with polynomials using integration
 - Find a general solution to a differential equation with trigonometric functions using integration
 - Find a general solution to a differential equation with exponential functions using integration
 - Find a general solution to a differential equation with logarithmic functions using integration
 - Initial Value Problems
-

- Verify a solution to an initial value problem of a linear differential equation
 - Verify a solution to a differential equation initial value problem with polynomials
 - Verify a solution to a differential equation initial value problem with trigonometric functions
 - Verify a solution to a differential equation initial value problem with exponential functions
 - Verify a solution to a differential equation initial value problem with logarithmic functions
- Find a solution to a linear differential equation initial value problem
 - Find a solution to a linear differential equation initial value problem with polynomials
 - Find a solution to a linear differential equation initial value problem with trigonometric functions
 - Find a solution to a linear differential equation initial value problem with exponential functions
 - Find a solution to a linear differential equation initial value problem with logarithmic functions
 - Identify a particular solution on a graph that satisfies an initial value problem
- Solve applications of linear differential equations
 - Solve motion applications with velocity using a differential equation initial value problem
 - Solve motion applications with position using a differential equation initial value problem

8.2 Direction Fields and Numerical Methods

- Create and Use a Direction Field
 - Introduction to direction fields
 - Find the slope of a solution curve at a given point
 - Plot the slope of a solution curve at a given point using technology
 - Use technology to sketch direction fields for a given differential equation
 - Use technology to sketch solution curves for a given initial-value problem
 - Equilibrium Solutions
 - Identify and classify equilibrium solutions using a direction field
 - Identify an equilibrium solution given a direction field
 - Classify an equilibrium solution as stable, unstable, or semi-stable
 - Euler's Method
 - Use Euler's Method to approximate the solution to a first-order differential equation
 - Estimate the solution to a differential equation involving a polynomial expression using Euler's Method
-

- Estimate the solution to a differential equation involving an exponential expression using Euler's Method
- Estimate the solution to a differential equation involving a trigonometric expression using Euler's Method
- Estimate the error in Euler's Method
 - Estimate the error in Euler's Method when applied to a differential equation involving a polynomial expression
 - Estimate the error in Euler's Method when applied to a differential equation involving an exponential expression
 - Estimate the error in Euler's Method when applied to a differential equation involving a trigonometric expression

8.3 Separable Equations

- Finding General Solutions Using Separation of Variables
 - Use the separation of variables technique to find constant solutions and set up integrals to solve differential equations
 - Find any constant solutions for differential equations
 - Use the separation of variables technique to set up the integral that will solve a differential equation
 - Use the separation of variables technique to set up the integral that will solve a differential equation when factoring is required
 - Find general solutions to differential equations using separation of variables
 - Find general solutions of differential equations with rational functions
 - Find general solutions of differential equations with polynomial functions using separation of variables
 - Find general solutions of differential equations with polynomial functions using separation of variables when factoring is required
 - Find general solutions of differential equations with trigonometric functions using separation of variables
 - Find general solutions of differential equations with exponential functions using separation of variables
 - Find general solutions of differential equations with logarithmic functions using separation of variables
 - Finding Particular Solutions and Solving Applications Using Separation of Variables
 - Find particular solutions to differential equations using separation of variables
 - Find particular solutions of differential equations with rational functions
 - Find particular solutions of differential equations with polynomial functions using separation of variables
 - Find particular solutions of differential equations with trigonometric functions using separation of variables
-

- Find particular solutions of differential equations with exponential functions using separation of variables
- Find particular solutions of differential equations with logarithmic functions using separation of variables
- Solve differential equations application problems using separation of variables
 - Solve Newton's Law of Cooling problems with separation of variables
 - Solve solution concentration problems with separation of variables

8.4 The Logistic Equation

- Population Growth and Carrying Capacity
 - Model population growth using a logistic differential equation given information about the population
 - Identify the carrying capacity and growth rate of a given logistic equation
 - Identify the values for which a logistic equation is increasing or decreasing
 - Write the equation for a population problem using the logistic equation
 - Use a logistic equation to predict population size
 - Find the solution to a logistic differential equation
 - Use the solution to a logistic differential equation to predict population size
 - Find an equation to model population given initial information
- Solving the Logistic Differential Equation
 - Sketch a direction field for the logistic equation
 - Draw a direction field for the logistic equation when given the growth rate and carrying capacity
 - Draw the solution curve for a logistic equation for a given initial population
 - Interpret the solution of a logistic equation
 - Use solutions of initial value problems to find population sizes
 - Use solutions of initial value problems to find the time it takes to reach a population level
 - Use the solution of a logistic equation to determine how long it will take to reach the carrying capacity
- Other Models of Population Growth
 - Using other models of population growth
 - Use the natural growth model
 - Use the Gompertz growth function
 - Use logistic models where the population becomes extinct
 - Use logistic models where the population is harvested

8.5 First-Order Linear Equations

- Writing a First-Order Linear Differential Equation in Standard Form
 - Determine whether a first-order differential equation is linear
 - Determine whether a differential equation with polynomial functions is linear
-

- Determine whether a differential equation with trigonometric functions is linear
- Determine whether a differential equation with logarithmic functions is linear
- Write a first-order linear differential equation in standard form
 - Rewrite a first-order differential equation with polynomial functions in standard form
 - Rewrite a first-order differential equation with trigonometric functions in standard form
 - Rewrite a first-order differential equation with logarithmic functions in standard form
 - Identify $P(x)$ and $Q(x)$ with polynomial functions
 - Identify $P(x)$ and $Q(x)$ with trigonometric functions
 - Identify $P(x)$ and $Q(x)$ with logarithmic functions
- Integrating Factors
 - Find an integrating factor
 - Find an integrating factor with polynomial functions
 - Find an integrating factor with trigonometric functions
 - Find an integrating factor with logarithmic functions
 - Use an integrating factor to solve first-order linear differential equations
 - Find the general solution to a first-order differential equation when the integrating factor is logarithmic
 - Find the general solution to a first-order differential equation when the integrating factor is trigonometric
 - Find the general solution to a first-order differential equation when the integrating factor is a polynomial
 - Find the solution to an initial-value first order differential equation when the integrating factor is a logarithmic
 - Find the solution to an initial-value first order differential equation when the integrating factor is trigonometric
 - Find the solution to an initial-value first order differential equation when the integrating factor is a polynomial
- Applications of First-Order Linear Differential Equations
 - Solve applications of first-order linear differential equations
 - Solve first-order differential equation problems related to electric circuits
 - Solve first-order differential equation problems related to volume mixing
 - Solve first-order differential equation problems related to free fall

Chapter 9: Sequences and Series

9.1 Sequences

- Finding Formulas for Sequences
-

- Write the terms of a sequence
 - Write the terms of an arithmetic sequence defined explicitly
 - Write the terms of a geometric sequence defined explicitly
 - Write the terms of a sequence defined explicitly
 - Write the terms of an arithmetic sequence defined recursively
 - Write the terms of a geometric sequence defined recursively
 - Write the terms of a sequence defined recursively
- Find the formula for the n th term of a sequence
 - Identify the explicit formula for the n th term of an infinite sequence
 - Identify the explicit formula for the n th term of an alternating infinite sequence
 - Write the explicit formula for the n th term of an infinite sequence
 - Write the explicit formula for the n th term of an alternating infinite sequence
 - Write the explicit formula for the n th term of a recursively defined sequence
- Determining the Limit of a Sequence
 - Determine the limit of a sequence graphically
 - Determine the limit of a sequence
 - Determine the limit of a sequence
 - Determine the limit of a sequence defined by a rational function
 - Determine the limit of a sequence defined by an exponential function
 - Determine the limit of a sequence defined by a trigonometric function
- Determining the Convergence of a Sequence
 - Determine whether a sequence converges or diverges
 - Determine whether a continuous function converges or diverges
 - Determine if a continuous function defined on a convergent sequence converges and if so find its limit
 - Determine if a sequence converges or diverges using L'Hospital's Rule
 - Determine the limit of a convergent sequence using the Squeeze Theorem
 - Use properties of bounded and monotonic sequences to determine convergence
 - Determine if a sequence is monotonic
 - Determine if a sequence is bounded
 - Determine convergence of a sequence using the Monotone Convergence Theorem

9.2 Infinite Series

- Determining Convergence of a Series
 - Determine whether a series converges or diverges
 - Find a partial sum of a given series
 - Determine if a series converges or diverges given the sequence of partial sums
 - Determine if a series converges or diverges using a sequence of partial sums
 - Determine the sum of a convergent series using algebraic properties
-

- Determine convergence of a geometric series
 - Determine if a series is geometric
 - Determine the common ratio of a geometric series
 - Determine if a geometric series converges
 - Determine the sum of a convergent geometric series
- Telescoping Series
 - Determine the convergence of a telescoping series
 - Determine if a series is telescoping
 - Determine if a telescoping series converges or diverges
 - Determine the sum of a telescoping series

9.3 The Divergence and Integral Tests

- Divergence and Integral Tests
 - Determine if a series diverges using the divergence test
 - Calculate an infinite limit
 - Calculate an infinite limit with a factorial expression
 - Use the divergence test for series
 - Use the divergence test for series with a factorial expression
 - Determine if a series converges or diverges using the integral test
 - Determine whether the integral test can be used for a given series
 - Determine the integral that can be used to represent a series for the integral test
 - Determine whether a series is convergent using the integral test
- The p -Series and Estimating Values of Series
 - Determine if a p -series converges or diverges
 - Identify the value of p for a given series
 - Determine whether a p -series is convergent
 - Estimate the value of a convergent series
 - Estimate the error for a given partial sum of a convergent series
 - Calculate the value of n that guarantees the remainder is within a certain error bound
 - Use a CAS to calculate a partial sum for a convergent series
 - Calculate the lower and upper bounds of a convergent series

9.4 Comparison Tests

- Comparison Tests
 - Use the comparison test to determine if a series converges or diverges
 - Select an appropriate series to use with the comparison test to show that a given series converges
 - Select an appropriate series to use with the comparison test to show that a given series diverges
-

- Use the comparison test to determine whether the given infinite series converges or diverges
- Use the limit comparison test to determine if a series converges or diverges
 - Calculate the infinite limit of a ratio of two series
 - Use the limit comparison test and a given geometric series to determine whether an infinite series converges or diverges
 - Use the limit comparison test to determine if a given infinite series converges or diverges
- Use a comparison test to determine an unknown value for which a series converges

9.5 Alternating Series

- Alternating Series
 - Determine whether an alternating series converges
 - Determine convergence of an infinite series using the Alternating Series Test for a series written as a partial sum
 - Determine convergence of an infinite series using the Alternating Series Test for a series written in summation notation
 - Determine the value of an unknown quantity in a series that makes the series converge
 - Determine the value of an unknown quantity in a series that makes the series diverge
 - Approximate the sum of an alternating series
 - Estimate the remainder of an alternating series
 - Use the alternating series remainder to approximate the sum of an alternating series
 - Use the alternating series remainder to find the number of terms required to approximate the sum of a series
 - Understand absolute convergence and conditional convergence
 - Identify a convergent series as having absolute convergence or conditional convergence
 - Determine whether a series is divergent, absolutely convergent, or conditionally convergent

9.6 Ratio and Root Tests

- Ratio and Root Tests
 - Determine absolute convergence of a series using the Ratio Test
 - Determine absolute convergence of a series using the Root Test
 - Determine the appropriate convergence test for a series
 - Identify whether a series is geometric, harmonic, p , or alternating
 - Identify the test that is appropriate for determining the convergence of a series
-

Chapter 10: Power Series

10.1 Power Series and Functions

- Power Series and Functions
 - Identify a power series
 - Determine the interval of convergence and radius of convergence of a power series
 - Determine the interval of convergence of a power series that has passed the ratio test
 - Determine the interval of convergence of a power series
 - Determine the radius of convergence of a power series
 - Represent a function using a power series
 - Use a power series to represent a function
 - Use a power series to represent a function not centered at 0
 - Determine the interval of convergence for functions represented by power series
 - Determine the interval of convergence for functions represented by power series not centered at 0

10.2 Properties of Power Series

- Combining Power Series
 - Combine power series
 - Combine power series by addition or subtraction
 - Determine the interval of convergence for a series combined by addition or subtraction
 - Multiply two power series together
 - Determine the interval of convergence of a series created by multiplying two power series
 - Create a new power series
 - Create a power series from a known power series
 - Create a power series from a known power series using partial fractions
 - Determine the interval convergence of a series that has been created by a combination of two series
 - Find the function represented by a given power series
 - Determine the interval of convergence of the function represented by a given power series
 - Differentiating and Integrating Power Series
 - Differentiate power series
 - Differentiate power series term-by-term
 - Determine convergence after differentiation
 - Evaluate the sum of a series
 - Integrate power series
 - Integrate power series term-by-term
-

- Determine convergence after integration

10.3 Taylor and Maclaurin Series

- Taylor and Maclaurin Series
 - Find a Taylor or Maclaurin polynomial for a function
 - Recognize if a given series is a Taylor series
 - Find a Taylor polynomial for a function at a given value
 - Find a Maclaurin polynomial for a function
 - Estimate a function value using a Taylor or Maclaurin polynomial
 - Estimate a function value using a Taylor polynomial
 - Estimate a function value using a Maclaurin polynomial
- Taylor's Theorem with Remainder
 - Use Taylor's Theorem with Remainder to answer questions regarding the error of a Taylor or Maclaurin series
 - Verify that a given degree of a Taylor polynomial would result in a desired accuracy
 - Determine the error of an estimated function value using Taylor's Theorem with Remainder
 - Determine the degree of a Maclaurin polynomial necessary for a desired accuracy
 - Determine the interval of convergence of a Taylor or Maclaurin series

10.4 Working with Taylor Series

- Binomial Series
 - Use a binomial series
 - Find the binomial series for a function
 - Estimate the value of a function using a binomial series
 - Use a binomial expansion to find the Taylor series of a function given a center
 - Functions Expressed as Taylor Series
 - Find the Taylor expansion of common functions
 - Find the Taylor series for an exponential function
 - Find the Taylor series for a logarithmic function
 - Find the Taylor series for a trigonometric function
 - Find the Taylor series for a given function using differentiation
 - Find the radius of convergence of a Maclaurin series for a logarithmic function
 - Find the radius of convergence of a Maclaurin series for a trigonometric function
 - Find the radius of convergence of a Maclaurin series for a binomial series
 - Applications of Taylor Series
 - Applications of Taylor series in integration
 - Use a Taylor series to evaluate limits
 - Use a Taylor series to evaluate nonelementary integrals
 - Use a Taylor series to estimate definite integrals within a given error
-

- Other applications of Taylor and power series
 - Use a Taylor series to approximate probability
 - Use a power series to solve a first order differential equation
 - Solve Airy's equation using a power series

Chapter 11: Parametric Equations and Polar Coordinates

11.1 Parametric Equations

- Graphs of Parametric Equations and Eliminating the Parameter
 - Graph parametric equations
 - Determine table values to use for drawing parametric curves by plotting points
 - Determine table values to use for drawing trigonometric parametric curves by plotting points
 - Identify a curve defined by parametric equations by plotting points
 - Identify a curve defined by trigonometric parametric equations by plotting points
 - Use technology to draw a curve defined by parametric equations
 - Identify the parametric equations of curves
 - Eliminate the parameter in parametric equations
 - Write the parametric equations of a curve in $y=f(x)$ form by eliminating the parameter in linear equations
 - Write the parametric equations of a curve in $y=f(x)$ form by eliminating the parameter in polynomial and radical equations
 - Write the parametric equations of a curve in $y=f(x)$ form by eliminating the parameter in exponential and logarithmic equations
 - Write the parametric equations of a curve in $y=f(x)$ form by eliminating the parameter in trigonometric equations
 - Determine the domain of a function whose parameter has been eliminated for linear equations
 - Determine the domain of a function whose parameter has been eliminated for trigonometric equations
 - Write parametric equations
 - Write the parametric equation of a curve
 - Write the parametric equations for a line segment given an orientation
 - Cycloids and Other Parametric Curves
 - Graph parametric equations for cycloids and hypocycloids
 - Use technology to graph a cycloid
 - Use technology to graph a hypocycloid
 - Determine the number of cusps on the curve of a hypocycloid
 - Solve applications using parametric equations
 - Model motion application problems with parametric equations
-

- Use parametric equations to find the maximum height of an object
- Use parametric equations to find the displacement of an object

11.2 Calculus of Parametric Curves

- Derivatives of Parametric Equations
 - Find the first derivative of a parametric curve
 - Determine the slope of a line defined parametrically without eliminating the parameter
 - Determine the derivative of a parametrically defined curve with linear functions
 - Determine the derivative of a parametrically defined curve with polynomial and radical functions
 - Determine the derivative of a parametrically defined curve with exponential and logarithmic functions
 - Determine the derivative of a parametrically defined curve with trigonometric functions
 - Determine the equation of the tangent line to a parametrically defined curve
 - Determine the slope of the tangent line at a given point
 - Find the points on a parametrically defined curve that have a given slope
 - Determine the equation of the tangent to a parametrically defined curve
 - Find the second derivative of a parametric curve
 - Determine the second derivative of a parametrically defined curve with polynomial and radical functions
 - Determine the second derivative of a parametrically defined curve with exponential and logarithmic functions
 - Determine the second derivative of a parametrically defined curve with trigonometric functions
 - Determine the concavity of a parametric curve
 - Determine the critical point(s) of a parametrically defined curve
 - Integrals Involving Parametric Equations
 - Find the area under a parametric curve
 - Set up a definite integral to describe the area under a parametric curve
 - Find the area under a parametric curve with linear functions
 - Use the equation for arc length for a parametric curve
 - Set up a definite integral to describe the arc length of a parametric curve
 - Use the equation for arc length for a parametric curve
 - Apply the formula for surface area to a volume generated by a parametric curve
 - Find the surface area of a known figure
 - Set up a definite integral to find the surface area of a shape generated by revolving a curve around the x-axis
-

- Use technology to find the surface area of a shape generated by revolving a curve around an axis

11.3 Polar Coordinates

- Defining Polar Coordinates
 - Locate points in a plane by using polar coordinates
 - Plot a point in polar coordinates
 - Determine the polar coordinates of a point plotted on a graph
 - Determine other representations of a given polar coordinate
 - Convert points between polar and rectangular coordinates
 - Convert coordinates from polar to rectangular form
 - Convert coordinates from rectangular to polar form
- Polar Equations
 - Convert equations between polar and rectangular form
 - Convert a polar equation to rectangular form
 - Convert a rectangular equation to polar form
 - Graph polar equations
 - Find zeros and maximum values for a polar equation
 - Create a table of values for a given polar equation
 - Identify the graph of a polar equation given a table of ordered pairs
 - Identify the basic shape of the graph of a polar equation
 - Convert a rectangular equation to polar form and graph the equation using technology
 - Convert a polar equation to rectangular form and graph the equation using technology
 - Identify symmetry

11.4 Area and Arc Length in Polar Coordinates

- The Calculus of Polar Coordinates
 - Apply the formula for the area of a region in polar coordinates
 - Determine the definite integral that represents the area under a polar curve
 - Determine the area of a region bounded by a polar curve
 - Determine intersection points of two polar curves
 - Determine the area of a region bounded by two polar curves
 - Solve application problems involving the derivative of polar equations
 - Find the slope of the tangent line to a polar curve at a specified point
 - Find the points on a polar curve where the tangent line is horizontal or vertical
 - Apply the formula for the arc length of a polar curve
 - Determine the definite integral that represents the arc length of a polar curve
 - Find the arc length of a curve
 - Use technology to approximate the length of a curve over a finite interval
-

11.5 Conic Sections

- Conic Sections in Standard Form
 - Identify and graph a parabola given in standard form
 - Write the equation of a parabola given its directrix and focus
 - Identify the key components of a parabola
 - Graph a parabola given in standard form
 - Write the equation of a parabola in standard form
 - Identify and graph an ellipse given in standard form
 - Write the equation of an ellipse given its key components
 - Identify the key components of an ellipse
 - Graph an ellipse given in standard form
 - Write the equation of an ellipse in standard form
 - Identify and graph a hyperbola given in standard form
 - Write the equation of a hyperbola given its foci and vertices
 - Identify the key components of a hyperbola
 - Graph a hyperbola given in standard form
 - Write the equation of a hyperbola in standard form
 - Eccentricity and Polar Equations of Conic Sections
 - Identify conic sections written in rectangular coordinates by eccentricity and write the equation of the conic
 - Identify conic sections written as polar equations by eccentricity and write the equation of the conic
 - Determine the eccentricity of a conic in polar coordinates
 - Recognize a conic section from its eccentricity value
 - Write the polar equation of a conic section given its eccentricity and directrix
 - Write the equation of a hyperbola given its foci and eccentricity
 - General Equations of Degree Two
 - Identify key components of conic sections given as polar equations and graph them
 - Identify the vertices of a conic section written as a polar equation
 - Identify the center of a conic section written as a polar equation
 - Identify the foci of a conic section written as a polar equation
 - Identify the directrices of a conic section written as a polar equation
 - Graph a conic section given as a polar equation
 - Identify a conic given in general form
 - Identify and graph a rotated conic section
 - Calculate the angle of rotation of axes for a conic
 - Rewrite the equation of a rotated conic.
 - Graph a rotated conic
 - Applications of Conic Sections
-

- Use applications of conic sections to solve problems
 - Use eccentricity to solve problems of planetary orbit
 - Use parabolas to solve application problems
 - Use ellipses to solve application problems

Chapter 12: Vectors in Space

12.1 Vectors in the Plane

- Introduction to Plane Vectors
 - Describe plane vectors
 - Describe a plane vector using correct notation
 - Determine if two vectors are equivalent vectors
 - Sketch a vector in two dimensions
 - Sketch a vector between points
 - Sketch a vector resulting from scalar multiplication
 - Sketch a vector resulting from vector addition
- Vectors in Component Form and Unit Vectors
 - Use vectors in component form
 - Express a vector in component form
 - Add or subtract vectors in component form
 - Perform a scalar multiplication on a vector in component form
 - Determine the magnitude of a vector
 - Find the component form of a vector using trigonometry
 - Express a vector in terms of unit vectors
 - Find a unit vector
 - Find a vector with a given magnitude and direction
 - Express a vector in terms of standard unit vectors
 - Perform operations using standard unit vectors
- Applications of Vectors
 - Solve applications using vectors
 - Find horizontal and vertical component vectors
 - Find a resultant force using vectors
 - Find the magnitude of a resultant force using vectors
 - Find the direction angle of a resultant force using vectors
 - Find a resultant velocity using vectors
 - Find the magnitude of a resultant velocity using vectors
 - Find the direction angle of a resultant velocity using vectors

12.2 Vectors in Three Dimensions

- The Three-Dimensional Coordinate System
 - Describe three dimensional space mathematically
-

- Identify a point in three-dimensional space
- Sketch a point in three-dimensional space
- Find the distance between two points in space
- Write equations in three dimensions
 - Write the equation of a plane parallel to a coordinate plane
 - Write the equation of a plane passing through three points in space
 - Find the equation of a sphere
 - Describe other equations in three dimensions
 - Graph other equations in three dimensions
- Vectors in Three Dimensions
 - Graph and find component forms for vectors in three dimensions
 - Sketch a vector in three-dimensional space
 - Express a vector in three dimensions in component form
 - Find an initial or terminal point on a vector
 - Express a vector in three dimensions in standard unit form
 - Perform vector operations in three dimensions
 - Perform operations on vectors in three dimensions
 - Determine the magnitude of a vector in three dimensions
 - Find a unit vector in three dimensions
 - Find vectors with a given magnitude and direction in space
- Applications of Vectors in Three Dimensions
 - Solve applications using vectors in three dimensions
 - Find a velocity vector in three dimensions
 - Find a force vector in three dimensions
 - Find the angle between a force and an axis

12.3 The Dot Product

- The Dot Product
 - Calculate the dot product
 - Calculate the dot product of two vectors in component form
 - Calculate the dot product of two vectors in unit vector form
 - Use properties of the dot product
 - Evaluate an expression containing the dot product of two vectors
 - Evaluate an expression containing the dot product of three vectors
 - Calculate the magnitude of a vector using the dot product
 - Angles Between Vectors and Projections of Vectors
 - Find angles using the dot product
 - Find the measure of the angle between two vectors
 - Identify the type of angle between two vectors
 - Determine whether two given vectors are orthogonal
-

- Determine if two vectors are parallel, orthogonal, or neither
- Determine the real number that makes two vectors orthogonal
- Find the angle a vector makes with a unit vector
- Find the direction cosines of a vector
- Find the direction angles of a vector
- Find the projection of one vector onto another vector
 - Find the scalar projection of one vector onto another vector
 - Find the projection of one vector onto another vector
 - Write the decomposition of a vector into orthogonal components
- Applications of the Dot Product
 - Solve applications of the dot product
 - Find velocity using scalar projection
 - Calculate the work done by a constant force

12.4 The Cross Product

- The Cross Product
 - Calculate the cross product
 - Calculate the cross product of two vectors
 - Identify a vector resulting from the cross product of vectors
 - Calculate the determinant of a matrix
 - Use a determinant to calculate the cross product of two vectors
 - Use properties of the cross product
 - Find the cross product of standard unit vectors
 - Use properties of the cross product to evaluate an expression
 - Calculate the magnitude of a cross product
 - Find a unit vector in the direction of a cross product
 - Find a vector orthogonal to the plane determined by three noncollinear points
 - Triple Scalar Products
 - Use properties of the triple scalar product
 - Calculate the triple scalar product
 - Determine if three vectors are coplanar
 - Determine if four points lie in the same plane
 - Area and Volume with the Cross Product
 - Determine areas and volumes using the cross product
 - Find the area of a parallelogram given adjacent sides
 - Find the area of a triangle given adjacent sides
 - Find the area of a triangle given three vertices
 - Find the distance from a point to a line
 - Find the volume of a parallelepiped determined by three vectors
 - Find the volume of a parallelepiped given adjacent edges
-

- Applications of the Cross Product
 - Solve applications using the cross product
 - Find torque about a point
 - Find the magnitude of the torque about a point
 - Find the magnitude of the force needed to supply a given torque
 - Determine the angle between a position vector and a force given a magnitude

12.5 Equations of Lines and Planes in Space

- Equations for a Line in Space
 - Find the parametric and symmetric equations of a line
 - Find the parametric and symmetric equations of a line through a point in a given direction
 - Find the parametric and symmetric equations of a line through two points
 - Find the parametric equations of a line segment
 - Use properties of lines in space
 - Find the point of intersection between two lines
 - Determine if two lines are equal, parallel but not equal, skew, or intersecting
- Equations for a Plane
 - Write the scalar and general form of the equations of a plane
 - Write the scalar and general form of the equation of a plane through a given point with a given normal
 - Write the scalar and general form of the equation of a plane given three points
 - Write the scalar and general form of the equations of a plane given a point and a line
 - Write the scalar and general form of the equations of a plane given two lines
 - Use properties of planes in space
 - Find the distance from a point to a given plane
 - Find the line of intersection between two planes
 - Find the angle between two planes
 - Find the distance between two parallel planes

12.6 Quadric Surfaces

- Cylinders and Quadric Surfaces
 - Identify and graph cylindrical surfaces
 - Identify and graph quadric surfaces
 - Determine the axis of symmetry of a quadric surface
 - Identify the equation or traces of a quadric surface
 - Graph the traces of a quadric surface
 - Identify the graph of a quadric surface
 - Identify equations of quadric surfaces
 - Identify the surface represented by a given equation
-

- Write the equation of a quadric surface in standard form

12.7 Cylindrical and Spherical Coordinates

- Cylindrical and Spherical Coordinates
 - Convert between cylindrical and rectangular coordinates
 - Convert from cylindrical to rectangular coordinates when given a point
 - Convert from rectangular to cylindrical coordinates when given a point
 - Find the equation of a surface in rectangular coordinates given the equation in cylindrical coordinates
 - Identify a surface in the cylindrical coordinate system
 - Find the equation of a surface in cylindrical coordinates given the equation in rectangular coordinates
 - Convert between spherical and rectangular coordinates
 - Convert from spherical to rectangular coordinates when given a point
 - Convert from rectangular to spherical coordinates when given a point
 - Find the equation of a surface in rectangular coordinates given the equation in spherical coordinates
 - Identify a surface in the spherical coordinate system
 - Find the equation of a surface in spherical coordinates given the equation in rectangular coordinates
 - Convert between cylindrical and spherical coordinates
 - Convert from cylindrical to spherical coordinates when given a point
 - Convert from spherical to cylindrical coordinates when given a point
- Applications with Cylindrical or Spherical Coordinates
 - Solve application problems involving cylindrical and spherical coordinates
 - Convert longitude and latitude to spherical coordinates
 - Represent an object with an appropriate coordinate system

Chapter 13: Vector-Valued Functions

13.1 Vector-Valued Functions and Space Curves

- Definition and Graphs of Vector-Valued Functions
 - Write equations of vector-valued functions
 - Determine the component functions for a vector-valued function
 - Write the equation of a vector-valued function
 - Eliminate the parameter of a vector-valued function
 - Write the equation for a transformation of a vector-valued function
 - Evaluate and determine the domain of a vector-valued function
 - Evaluate a vector-valued function
 - Determine the domain of a vector-valued function
 - Graph vector-valued functions
-

- Graph projections of a vector-valued curve on coordinate planes
- Graph a vector-valued function in two dimensions
- Graph a vector-valued function in three dimensions
- Limits and Continuity of Vector-Valued Functions
 - Determine limits and continuity of vector-valued functions
 - Calculate the limit of a vector-valued function
 - Determine continuity of a vector-valued function
- Intersections of Surfaces
 - Determine the intersection of two surfaces
 - Find a vector-valued function that represents the intersection of two surfaces
 - Sketch the space curve represented from the intersection of two surfaces

13.2 Calculus of Vector-Valued Functions

- Derivatives of Vector-Valued Functions
 - Find the derivative of vector-valued functions
 - Find the derivative of a vector-valued function
 - Find a higher-order derivative of a vector-valued function
 - Use linear properties of the derivative of vector-valued functions
 - Use the scalar multiple property of the derivative of vector-valued functions
 - Use the sum and difference properties of the derivative of vector-valued functions
 - Use the scalar product property of the derivative of vector-valued functions
 - Use dot product, cross product, and chain rule properties of derivatives of vector-valued functions
 - Use the dot product property of the derivative of vector-valued functions
 - Use the cross product property of the derivative of vector-valued functions
 - Use the chain rule property of the derivative of vector-valued functions
- Tangent Vectors and Unit Tangent Vectors
 - Compute tangent and unit tangent vectors for vector-valued functions
 - Find a tangent vector for a vector-valued function at a specified point
 - Find a unit tangent vector for a vector-valued function
 - Find a unit tangent vector for a vector-valued function at a specified point
 - Find an equation of a line tangent to a vector-valued function at a specified point
- Integrals of Vector-Valued Functions
 - Find integrals of vector-valued functions
 - Find the antiderivative of a vector-valued function given an initial condition
 - Evaluate the indefinite integral of a vector-valued function
 - Evaluate a definite integral of a vector-valued function

13.3 Arc Length and Curvature

- Arc Length and Arc-Length Parameterization
-

- Find the arc length for a vector-valued function
 - Write an integral to find the arc length of a curve
 - Find the arc length of a curve
- Find the arc length parametrization for a vector-valued function
 - Write an integral that represents the arc length function for a curve
 - Find the arc length function for a curve
 - Identify whether a vector-valued function is parametrized by arc length
 - Find the arc length parametrization of a curve given an arc length function
 - Find the arc length parametrization of a curve
- Curvature
 - Find the curvature of a vector-valued function
 - Find the curvature of a curve using the arc length parametrization
 - Find the curvature of a curve using a different parametrization
 - Find the curvature for a space curve using a cross-product
 - Find the curvature for a plane curve expressed as $y(x)$
 - Identify where the curvature of a plane curve is a minimum or maximum
- Normal and Binormal Vectors
 - Find the normal and binormal vectors for a vector-valued function
 - Find the principal unit normal vector for a curve
 - Find the binormal vector for a curve
 - Find the osculating circle and radius of curvature for a curve
 - Find the radius of curvature for a curve
 - Find the equation of the osculating circle for a plane curve at a point
 - Find the normal and osculating planes for a space curve at a point

13.4 Motion in Space

- Motion Vectors in the Plane and in Space
 - Find the velocity and acceleration vectors of particle in motion
 - Find the velocity vector of a particle moving along a curve
 - Find the speed of a particle moving along a curve
 - Identify when the speed of a moving particle is a maximum or minimum
 - Find the acceleration vector of a particle moving along a curve
 - Solve an initial value problem to find the position and velocity vectors given the acceleration vector
 - Use technology to graph the velocity and acceleration vectors for a moving particle
 - Components of the Acceleration Vector
 - Find the tangential and normal components of acceleration
 - Find the tangential component of the acceleration vector of a moving particle
 - Find the normal component of the acceleration vector of a moving particle
-

- Decompose the acceleration vector into its tangential and normal components
- Solve problems involving centripetal force and friction
- Projectile Motion
 - Solve problems involving projectile motion
 - Find the maximum height of a projectile in motion
 - Find the horizontal range of a projectile in motion
 - Find the time at which a projectile attains a given position
 - Find the initial velocity needed for a projectile to attain a given position
 - Use Kepler's laws of planetary motion to solve problems

Chapter 14: Differentiation of Functions of Several Variables

14.1 Functions of Several Variables

- Functions of Two Variables
 - Recognize a function of two variables and identify its domain and range
 - Evaluate a function of two variables
 - Find the domain of a function of two variables
 - Find the range of a function of two variables
 - Sketch a graph of a function of two variables
 - Identify the graph of a function of two variables
 - Find the equation for the level curve of a function of two variables
 - Find the equation for the vertical trace of a function of two variables
 - Sketch the level curves of a function of two variables
 - Sketch the vertical traces of a function of two variables
- Functions of More Than Two Variables and Applications
 - Recognize a function of three or more variables and identify its level surfaces
 - Evaluate a function of three variables
 - Find the domain of a function of three variables
 - Find the equation for a level surface of a function of three variables
 - Identify the graph for a level surface of a function of three variables
 - Use functions of several variables in applications
 - Evaluate a function of two variables in applications
 - Find the equation for a level curve of a function of two variables in applications
 - Find the equation for a level surface of a function of three variables in applications

14.2 Limits and Continuity

- Limit of a Function of Two Variables
 - Use the formal definition of a limit of a function of two variables
 - Given a limit statement for a function of two variables, write the corresponding epsilon-delta statement
 - Given the epsilon-delta statement of a limit of a function of two variables, write the corresponding limit statement
 - Evaluate the limit of a function of two variables
 - Use the sum, difference, or constant multiple laws to evaluate the limit of a function of two variables
-

- Use the product or quotient laws to evaluate the limit of a function of two variables
- Use the power or root laws to evaluate the limit of a function of two variables
- Use algebraic techniques to evaluate the limit of a function of two variables
- Evaluate the limit of a function of two variables at a boundary point
 - Understand the difference between interior and boundary points
 - Evaluate the limit of a function of two variables at a boundary point
- Use different paths of approach to determine if the limit of a function of two variables exists
 - Evaluate the limit of a function of two variables for a particular path of approach
 - Use paths of approach to determine if the limit of a function of two variables exists
- Continuity of a Function of Two Variables
 - Understand the conditions for continuity of a function of two variables
 - Given a graph of a function of two variables, determine why the definition of continuity does not hold
 - Verify the continuity of a function of two variables at a point
 - Determine the region of the coordinate plane where a function of two variables is continuous
 - Apply the theorems of continuous functions
 - Apply the sum theorem for continuity of functions of two variables
 - Apply the product theorem for continuity of functions of two variables
 - Apply the composition theorem for continuity of functions of two variables
- Limit and Continuity of a Function of More Than Two Variables
 - Verify the continuity of the function of three variables at a point
 - Evaluate the limit of a function of three variables
 - Verify the continuity of a function of three variables at a point

14.3 Partial Derivatives

- Partial Derivatives of a Function
 - Calculate the partial derivatives of a function of two variables
 - Calculate the sign of a partial derivative using the graph of a surface
 - Calculate a first-order partial derivative of a function of two variables
 - Calculate a first-order partial derivative of a function of two variables at a point
 - Use contours to estimate a partial derivative of a function at a given point
 - Determine the slope of a surface at a point
 - Determine points in which first-order partial derivatives are equal for a given function
 - Calculate the partial derivatives of a function of more than two variables
 - Calculate a first-order partial derivative of a function of more than two variables
 - Calculate a first-order partial derivative of a function of more than two variables at a point
 - Determine the rate of change of a function with respect to a given variable
 - Higher-Order Partial Derivatives and Partial Differential Equations
 - Determine the higher-order derivatives of a function of two or more variables
 - Calculate second-order partial derivatives of a function of two variables
-

- Calculate higher-order partial derivatives of a function of two or more variables
- Verify a solution to a partial differential equation
 - Verify a solution to a partial differential equation with two variables
 - Verify a solution to a partial differential equation with more than two variables

14.4 Tangent Planes and Linear Approximation

- Tangent Planes
 - Find the normal line to a surface at a point
 - Find the normal vector to a surface at a point
 - Find parametric or symmetric equations for the normal line to a surface at a point
 - Find the tangent plane to a surface at a point
 - Find the equation of the tangent plane to a surface at a point
 - Find the points on a surface where the tangent plane is horizontal
 - Find the angle of inclination of the tangent plane to a surface at a point
- Linear Approximations of Functions of Several Variables
 - Find the linear approximation of a function of two or more variables at a point
 - Find the linear approximation of a function of two variables at a point
 - Use the linear approximation of a function of two variables to estimate a function value
 - Find the linear approximation of a function of more than two variables at a point
 - Use the linear approximation of a function of more than two variables to estimate a function value
- Differentiability of a Function of Several Variables
 - Show that a function of several variables is differentiable
 - Show that a function of two variables is differentiable using the definition
 - Determine whether a function of two variables is differentiable using its partial derivatives
 - Determine whether a function of two variables is continuous
 - Determine whether a function of more than two variables is differentiable
 - Determine whether a function of more than two variables is continuous
- Differentials and Maximum Error
 - Find the differential of a function of two or more variables
 - Find the total differential of a function of two variables
 - Use a differential to estimate a change in a function of two variables
 - Find the total differential of a function of more than two variables
 - Use a differential to estimate a change in a function of more than two variables
 - Use a differential to estimate a change in a geometric application
 - Use a differential to estimate a change in area or surface area
 - Use a differential to estimate a change in volume
 - Use a differential to estimate the error introduced by an error in measurement
 - Use a differential to estimate the propagated error
 - Use a differential to estimate the relative or percent error

14.5 The Chain Rule

- The Chain Rule for One or Two Independent Variables
 - Use the chain rule for one independent variable
-

- Calculate the derivative of a function with one independent variable using the chain rule
- Find the rate of change of a function with one independent variable involving a real-world application
- Use the chain rule for several independent variables
 - Calculate a partial derivative of a function with two independent variables using the chain rule
 - Calculate a partial derivative of a function with several independent variables using the generalized chain rule
- Implicit Differentiation by Partial Derivatives
 - Use implicit differentiation to calculate partial derivatives
 - Use implicit differentiation to calculate the derivative of an implicitly defined function of two variables
 - Use implicit differentiation to calculate the partial derivative of an implicitly defined function of three variables

14.6 Directional Derivatives and the Gradient

- Directional Derivatives and Gradients for Functions in Two Variables
 - Find a directional derivative for a function of two variables
 - Find a directional derivative of a function of two variables using the limit definition
 - Find a directional derivative of a function of two variables in the direction of a given angle
 - Find a directional derivative of a function of two variables in the direction of a given vector
 - Find a directional derivative of a function of two variables at a point in the direction of another point
 - Find and use the gradient of a function of two variables
 - Find the gradient of a function of two variables
 - Use the gradient to find a directional derivative for a function of two variables
 - Applications of Directional Derivatives and Gradients
 - Use the gradient to maximize or minimize a directional derivative for a function of two variables
 - Find the maximum or minimum value of a directional derivative of a function of two variables
 - Find the direction in which a directional derivative is maximized or minimized for a function of two variables
 - Find normal and tangent vectors to a level curve of a function of two variables
 - Find a normal vector to a level curve of a function of two variables
 - Find a tangent vector to a level curve of a function of two variables
 - Directional Derivatives and Gradients for Functions in Three Variables
 - Find and use the gradient of a function of three variables
 - Find the gradient of a function of three variables
 - Find a directional derivative of a function of three variables
 - Use the gradient to maximize or minimize a directional derivative for a function of three variables
-

- Find the maximum or minimum value of a directional derivative of a function of three variables
- Find the direction in which a directional derivative is maximized or minimized for a function of three variables
- Applications of Functions of Two or More Variables
 - Solve application problems for functions of two or more variables
 - Solve problems involving temperature for a function of two variables
 - Solve problems involving temperature for a function of three variables
 - Solve problems involving topographical maps for a function of two variables
 - Solve problems involving topographical maps for a function of three variables
 - Solve problems involving electrical potential for a function of two or more variables
 - Solve problems involving electrical potential for a function of three variables

14.7 Maxima/Minima Problems

- Critical Points and the Second Derivative Test for Functions of Several Variables
 - Use partial derivatives to locate critical points of a function of two variables
 - Find the critical points of a polynomial function using partial derivatives
 - Find the critical points of a radical function using partial derivatives
 - Find the critical points of an exponential function using partial derivatives
 - Use the second derivative test to classify critical points of a function of two variables
 - Given a critical point and a function, classify the point as a local maximum, a local minimum, or a saddle point
 - Use the second derivative test to find the local extrema of a given function
 - Find the absolute extrema of a function over a closed, rectangular region
 - Find the absolute extrema of a function over a closed disk
- Maximization and Minimization Problems for Functions of Several Variables
 - Solve maximization and minimization problems that involve functions of several variables
 - Find three positive numbers that satisfy given conditions
 - Find points on a plane or surface closest to a given point
 - Determine the maximum or minimum volume of a given object
 - Solve maximum revenue and profit problems

14.8 Lagrange Multipliers

- Lagrange Multipliers
 - Use the method of Lagrange multipliers to optimize a function of two variables
 - Graphically identify maximum and minimum values of a function of two variables given a constraint
 - Use the method of Lagrange multipliers to optimize a function of two variables with one constraint
 - Determine the absolute maximum and minimum values of a function using Lagrange multipliers on the boundary
 - Use the method of Lagrange multiples to optimize a function of three variables
 - Use the method of Lagrange multiples to optimize a function of three variables with one constraint
 - Use the method of Lagrange multiples to optimize a function of three variables with two constraints
-

- Applications Using Lagrange Multipliers
 - Use the method of Lagrange multipliers to solve optimization problems with geometric applications
 - Use the method of Lagrange multipliers to optimize the distance between geometric objects
 - Use the method of Lagrange multipliers to optimize a geometric feature of an inscribed figure
 - Use the method of Lagrange multipliers to optimize one geometric feature given a constraint on another
 - Use the method of Lagrange multipliers to find the highest point on the intersection of two surfaces
 - Use the method of Lagrange multipliers to solve optimization problems with business applications
 - Use the method of Lagrange multipliers to minimize a cost function
 - Use the method of Lagrange multipliers to maximize a profit function
 - Use the method of Lagrange multipliers to optimize production level

Chapter 15: Multiple Integration

15.1 Double Integrals over Rectangular Regions

- Double Riemann Sums
 - Use a double Riemann sum to approximate a double integral over a rectangular region
 - Use a double Riemann sum to approximate a double integral given a table of values for a function
 - Use a double Riemann sum to approximate a double integral given the level curves for a function
 - Use a double Riemann sum to approximate a double integral of a polynomial function
 - Use a double Riemann sum to approximate a double integral of a trigonometric function
 - Fubini's Theorem
 - Use Fubini's theorem to set up an iterated double integral over a rectangular region
 - Set up an iterated double integral of a polynomial function over a rectangular region
 - Set up an iterated double integral of a radical function over a rectangular region
 - Set up an iterated double integral of a trigonometric function over a rectangular region
 - Set up an iterated double integral of an exponential function over a rectangular region
 - Set up an iterated double integral of a logarithmic function over a rectangular region
 - Set up an iterated double integral of an inverse trigonometric function over a rectangular region
 - Use Fubini's theorem to evaluate an iterated double integral over a rectangular region
 - Evaluate an iterated double integral of a polynomial function over a rectangular region
-

- Evaluate an iterated double integral of a radical function over a rectangular region
 - Evaluate an iterated double integral of a trigonometric function over a rectangular region
 - Evaluate an iterated double integral of a exponential function over a rectangular region
 - Evaluate an iterated double integral of a logarithmic function over a rectangular region
 - Evaluate an iterated double integral of an inverse trigonometric function over a rectangular region
 - Switching the Order of Integration
 - Change the order of integration of an iterated double integral over a rectangular region
 - Change the order of integration of an iterated double integral over a rectangular region
 - Change the order of integration to evaluate a double integral of a polynomial function
 - Change the order of integration to evaluate a double integral of a radical function
 - Change the order of integration to evaluate a double integral of an exponential function
 - Properties of Double Integrals
 - Understand the properties of double integrals
 - Evaluate a linear combination of double integrals over a rectangular region
 - Evaluate a double integral over a union of adjacent rectangular regions
 - Evaluate a double integral of a polynomial function that can be written as a product of two integrals
 - Evaluate a double integral of a radical function that can be written as a product of two integrals
 - Evaluate a double integral of a trigonometric function that can be written as a product of two integrals
 - Evaluate a double integral of an exponential function that can be written as a product of two integrals
 - Evaluate a double integral of a logarithmic function that can be written as a product of two integrals
 - Find upper and lower bounds on the value of a double integral
 - Find upper and lower bounds on the value of a double integral of a polynomial function
 - Find upper and lower bounds on the value of a double integral of a trigonometric function
 - Find upper and lower bounds on the value of a double integral of an exponential function
 - Applications of Double Integrals Over Rectangular Regions
 - Use a double Riemann sum to approximate the volume of a solid with a rectangular base
 - Use a double Riemann sum to approximate the volume of a solid with a rectangular base given a table of values
-

- Use a double Riemann sum to approximate the volume of a solid with a rectangular base bounded above by a polynomial function
- Use a double Riemann sum to approximate the volume of a solid with a rectangular base bounded above by a trigonometric function
- Use a double integral to determine the volume of a solid with a rectangular base
 - Use a double integral to determine the volume of a solid with a rectangular base bounded above by a plane
 - Use a double integral to determine the volume of a solid with a rectangular base bounded above by an elliptic paraboloid
 - Use a double integral to determine the volume of a solid with a rectangular base bounded above by a trigonometric function
 - Use a double integral to determine the volume of a solid with a rectangular base bounded above by an exponential function
- Average Value of a Function of Two Variables
 - Use a double Riemann sum to approximate the average value of a function of two variables over a rectangular region
 - Use a double Riemann sum to approximate the average value of a function over a rectangular region given a table of values
 - Use a double integral to determine the average value of a function of two variables over a rectangular region
 - Use a double integral to determine the average value of a polynomial function over a rectangular region
 - Use a double integral to determine the average value of a trigonometric function over a rectangular region
 - Use a double integral to determine the average value of an exponential function over a rectangular region
 - Use a double integral to determine the average value of an inverse trigonometric function over a rectangular region
 - Use a double integral to determine the average squared distance from a point to a rectangular region

15.2 Double Integrals over General Regions

- General Regions of Integration
 - Recognize when a function of two variables is integrable over a general region
 - Determine whether a region bounded by linear functions is of Type I or Type II
 - Determine whether a region bounded by two quadratic functions is of Type I or Type II
 - Determine whether a region bounded by two trigonometric functions is of Type I or Type II
 - Determine whether a region bounded by a trigonometric and a polynomial function is of Type I or Type II
 - Determine whether a region bounded by a quadratic and a radical function is of Type I or Type II
 - Set up an iterated double integral for a function of two variables over a general region
 - Set up an iterated double integral of a polynomial function over a region of Type I

- Set up an iterated double integral of an exponential or logarithmic function over a region of Type I
 - Set up an iterated double integral of a polynomial function over a region of Type II
 - Set up an iterated double integral of an exponential or logarithmic function over a region of Type II
 - Double Integrals Over Non-Rectangular Regions
 - Evaluate a double integral using Fubini's theorem
 - Evaluate a double integral over a polynomial function region of Type I
 - Evaluate a double integral over a trigonometric function region of Type I
 - Evaluate a double integral over an inverse trigonometric function region of Type II
 - Evaluate a double integral over a polynomial and radical function region of Type II
 - Evaluate a double integral over a polynomial function region of Type II
 - Double Integrals by Decomposing Regions
 - Evaluate a double integral over a complex region by decomposing the region
 - Evaluate a double integral of a polynomial function over a more complex exponential function and polynomial function region using a union of regions of Type I
 - Evaluate a double integral of a polynomial function over a more complex polynomial function region using a union of regions of Type I
 - Evaluate a double integral of a polynomial function over a more complex polynomial function region using a union of regions of Type II
 - Evaluate a double integral of a polynomial function over a complex triangular region using a union of regions of Type I
 - Evaluate a double integral of a polynomial function over a complex quadrilateral region using a union of regions of Type I
 - Evaluate a double integral of a polynomial function over a complex parabolic region using a union of regions of Type I
 - Evaluate a double integral of an exponential function over an algebraic region using a union of regions of Type I
 - Evaluate a double integral a polynomial function over a more complex polynomial function region using a union of regions of Type II
 - Double Integrals by Changing the Order of Integration
 - Simplify the calculation of an iterated integral by changing the order of integration
 - Reverse the order of integration for an iterated double integral over a polynomial function region
 - Reverse the order of integration for an iterated double integral over a triangular region
 - Evaluate a double integral of a polynomial function over a polynomial region by using the easier order of integration
 - Evaluate a double integral of a polynomial function over a parabolic region by using the easier order of integration
-

- Evaluate a double integral of a rational function over an exponential function and polynomial function region by using the easier order of integration
 - Evaluate a double integral of a polynomial function over a trigonometric region by using the easier order of integration
 - Evaluate a double integral of a trigonometric function by changing the region from Type I to Type II
 - Evaluate a double integral of a polynomial function over a region defined by radical functions by changing the region from Type II to Type I
 - Evaluate a double integral of a trigonometric function by changing the region from Type II to Type I
 - Calculating Volumes with Double Integrals
 - Set up an iterated double integral to calculate the volume of a region between two surfaces
 - Set up an iterated double integral to find the volume of a solid in the first octant under a plane
 - Set up an iterated double integral to find the volume bounded by two cylinders and two planes
 - Use double integrals to calculate the volume of a region between two surfaces
 - Use a double integral to find the volume of a solid in the first octant under a plane
 - Use a double integral to find the volume of a solid between a planar region and a sphere
 - Calculating Areas with Double Integrals
 - Set up an iterated double integral to calculate the area of a general plane region
 - Set up an iterated double integral to find the area of a region bounded by two polynomial functions
 - Use double integrals to calculate the area of a general plane region
 - Use a double integral to find the area of a region bounded by two polynomial functions by decomposing the region
 - Use a double integral to find the area of a region bounded by two trigonometric functions
 - Use a double integral to find the area of a region bounded by two logarithmic functions
 - Calculating Average Values with Double Integrals
 - Find the average value of a function over a general region
 - Set up an iterated double integral to find the average value of a linear function on a triangular region
 - Set up an iterated double integral of a polynomial function to find the average value of a function over a region bounded by a parabola and a line
 - Use a double integral to find the average value of a polynomial function over a region bounded by two radical functions
 - Improper Double Integrals
 - Evaluate a double improper integral
 - Use Fubini's theorem to evaluate an improper double integral of an exponential function over a polynomial region
-

- Use Fubini's theorem to evaluate an improper double integral of a radical function over a circular region
- Use Fubini's theorem to evaluate an improper double integral of a rational function over a radical region
- Evaluate an improper integral of a rational function over an unbounded region
- Evaluate an improper double integral of a rational function over an unbounded region
- Evaluate an improper double integral of an exponential function over an unbounded region
- Applications of Double Integrals in Probability
 - Find the probability and the expected value given a joint density function
 - Use a double integral to find a probability given an exponential joint density function
 - Use a double integral to find a probability given a constant joint density function
 - Use a double integral to find the expected value of Y given a polynomial joint density function
 - Use a double integral to find the expected value of X given a polynomial joint density function
 - Use a double integral to find the expected time given an exponential joint density function

15.3 Double Integrals in Polar Coordinates

- Double Integrals over Polar Rectangular Regions
 - Express a region of the plane in terms of polar coordinates
 - Express a region represented by a general description in polar coordinates
 - Express a region represented by boundary equations or inequalities in polar coordinates
 - Express a region represented by a graph in polar coordinates
 - Evaluate a double integral over a polar rectangular region
 - Set up an iterated integral of an algebraic function over a polar rectangular region
 - Set up an iterated integral of a transcendental function over a polar rectangular region
 - Evaluate an iterated integral of an algebraic function over a polar rectangular region
 - Evaluate an iterated integral of a transcendental function over a polar rectangular region
 - Converting Double Integrals in Rectangular Coordinates to Polar Coordinates
 - Convert a double integral in rectangular coordinates to polar coordinates
 - Set up an iterated integral of a polynomial function by converting to polar coordinates
 - Evaluate an iterated integral by first converting to polar coordinates
 - Convert an integral of a polynomial function from rectangular to polar coordinates over a polar rectangle
 - Convert an integral of a trigonometric function from rectangular to polar coordinates over a polar rectangle
-

- Convert an integral of a polynomial function from rectangular to polar coordinates over a polar rectangle and evaluate
- Convert an integral of a trigonometric function from rectangular to polar coordinates over a polar rectangle and evaluate
- Double Integrals Over General Polar Regions
 - Evaluate a double integral over a general polar region
 - Set up a double integral in polar coordinates over a general polar region
 - Evaluate a double integral in polar coordinates over a general polar region
 - Convert an integral of a polynomial function or radical function from rectangular to polar coordinates over a general polar region
 - Convert an integral of a rational function from rectangular to polar coordinates over a general polar region
 - Convert an integral of a rational function from rectangular to polar coordinates over a general polar region and evaluate
- Polar Areas and Volumes
 - Use double integrals in polar coordinates to calculate areas
 - Use double integrals in polar coordinates to compute the area of a region bounded by a polar curve
 - Use double integrals in polar coordinates to compute the area of a region bounded between polar curves
 - Use double integrals in polar coordinates to calculate volumes
 - Set up a double integral in polar coordinates to compute the volume of a region bounded by a surface and a plane
 - Use double integrals in polar coordinates to compute the volume of a region bounded by a surface and a plane
 - Use double integrals in polar coordinates to compute the volume of a region bounded by two surfaces
- Improper Double Integrals in Polar Coordinates
 - Evaluate a double improper integral in polar coordinates
 - Use double integrals in polar coordinates to compute an improper integral of an exponential function

15.4 Triple Integrals

- Triple Integrals Over a Rectangular Box
 - Evaluate a triple integral over a rectangular box
 - Evaluate a triple iterated integral of a polynomial function over a rectangular box
 - Evaluate a triple iterated integral of a rational function over a rectangular box
 - Evaluate a triple iterated integral of a trigonometric function over a rectangular box
 - Evaluate a triple iterated integral of an exponential function over a rectangular box
 - Evaluate a triple iterated integral of a logarithmic function over a rectangular box
 - Triple Integrals Over a General Bounded Region
 - Evaluate a triple integral over a general region that has a rectangular projection
 - Evaluate a triple integral of an algebraic function of Type 1 over a general region that has a rectangular projection
-

- Evaluate a triple integral of Type 2 over a general region that has a rectangular projection
- Evaluate a triple integral of Type 3 over a general region that has a rectangular projection
- Evaluate a triple integral over a general region that has a non-rectangular projection
 - Evaluate a triple integral of Type 1 over a general region that has a non-rectangular projection
 - Evaluate a triple integral of Type 2 over a general region that has a non-rectangular projection
 - Evaluate a triple integral of Type 3 over a general region that has a non-rectangular projection
- Evaluate a triple integral over a general region given a description of the region
 - Set a triple integral over a general region given a description of the region
 - Evaluate a triple integral over a general region given a description of the region
- Changing the Order of Integration of Triple Integral
 - Rewrite a triple integral over a rectangular box by changing the order of integration
 - Rewrite a triple integral of an algebraic function over a rectangular box by changing the order of integration
 - Rewrite a triple integral of a transcendental function over a rectangular box by changing the order of integration
 - Evaluate a triple integral over a rectangular box by changing the order of integration
 - Evaluate a triple integral of a polynomial function over a rectangular box by changing the order of integration
 - Evaluate a triple integral of a rational function over a rectangular box by changing the order of integration
 - Evaluate a triple integral of a trigonometric function over a rectangular box by changing the order of integration
 - Evaluate a triple integral of an exponential function over a rectangular box by changing the order of integration
- Volume and Average Value of a Function of Three Variables
 - Use a triple integral to find a volume of a solid bounded region
 - Set up a triple integral to find the volume of a solid
 - Evaluate a triple integral to find the volume of a solid
 - Use a triple integral to find the average value of a function of three variables
 - Use a triple integral to find the average value of an algebraic function of three variables
 - Use a triple integral to find the average value of a transcendental function of three variables

15.5 Triple Integrals in Cylindrical and Spherical Coordinates

- Integration in Cylindrical Coordinates
 - Evaluate a triple integral in cylindrical coordinates
 - Evaluate a triple integral over a cylindrical box
 - Set up a triple integral in cylindrical coordinates over a general region
 - Convert a triple integral in rectangular coordinates to cylindrical coordinates
 - Convert a triple integral in rectangular coordinates to cylindrical coordinates
-

- Convert a triple integral in rectangular coordinates to cylindrical coordinates and evaluate
- Integration in Spherical Coordinates
 - Evaluate a triple integral in spherical coordinates
 - Evaluate a triple integral over a spherical box
 - Set up a triple integral over a spherical box
 - Convert a triple integral in rectangular coordinates to spherical coordinates
 - Convert a triple integral of a polynomial function in rectangular coordinates to spherical coordinates
 - Convert a triple integral of a radical function in rectangular coordinates to spherical coordinates
 - Convert a triple integral of a polynomial function in rectangular coordinates to spherical coordinates and evaluate
 - Convert a triple integral of a radical function in rectangular coordinates to spherical coordinates and evaluate
- Volume in Cylindrical Coordinates
 - Find the volume of a solid using a triple integral in cylindrical coordinates
 - Set up a triple integral in cylindrical coordinates to find the volume of a solid whose boundaries are given in rectangular coordinates
 - Find the volume of a solid using a triple integral in cylindrical coordinates whose boundaries are given in rectangular coordinates
- Volume in Spherical Coordinates
 - Find the volume of a solid using a triple integral in spherical coordinates
 - Find the volume of a solid using a triple integral in spherical coordinates
 - Find the volume of a solid using a triple integral in spherical coordinates whose boundaries are given in spherical coordinates
 - Set up a triple integral in spherical coordinates to find the volume of a solid whose boundaries are given in rectangular coordinates
 - Find the volume of a solid using a triple integral in spherical coordinates whose boundaries are given in rectangular coordinates
 - Find the volume of a solid between two surfaces using a triple integral in spherical coordinates
 - Set up a triple integral in spherical coordinates to find the volume between two surfaces given in rectangular coordinates
 - Find the volume between two surfaces given in rectangular coordinates using a triple integral in spherical coordinates

15.6 Calculating Centers of Mass and Moments of Inertia

- Mass in Two Dimensions
 - Use double integrals to find the mass of a lamina
 - Use double integrals to find the mass of a rectangular lamina
 - Use double integrals to find the mass of a triangular lamina
 - Use double integrals to find the mass of a trapezoidal lamina
 - Use double integrals to find the mass of a circular lamina
 - Use double integrals to find the mass of an elliptical lamina
 - Center of Mass in Two Dimensions
-

- Use double integrals to find the moments of a lamina
 - Use double integrals to find the moments of a rectangular lamina
 - Use double integrals to find the moments of a triangular lamina
 - Use double integrals to find the moments of a trapezoidal lamina
 - Use double integrals and technology to find the moments of a circular lamina
 - Use double integrals to find the moments of an elliptical lamina
- Use double integrals to find the center of mass of a lamina
 - Use double integrals to find the center of mass of a rectangular lamina
 - Use double integrals to find the center of mass of a triangular lamina
 - Use double integrals to find the center of mass of a trapezoidal lamina
 - Use double integrals and technology to find the center of mass of a circular lamina
 - Use double integrals to find the center of mass of an elliptical lamina
- Use double integrals to find the centroid of a lamina
 - Use double integrals to find the centroid of a triangular lamina
 - Use double integrals to find the centroid of a trapezoidal lamina
 - Use double integrals to find the centroid of an elliptical lamina
- Moments of Inertia in Two Dimensions
 - Use double integrals to find the moments of inertia of a lamina
 - Use double integrals to find the moments of inertia of a rectangular lamina
 - Use double integrals to find the moments of inertia of a triangular lamina
 - Use double integrals to find the moments of inertia of a trapezoidal lamina
 - Use double integrals and technology to find the moments of inertia of a circular lamina
 - Use double integrals and technology to find the moments of inertia of an elliptical lamina
 - Use double integrals to find the radius of gyration of a lamina
 - Use double integrals to find the radius of gyration of a rectangular lamina
 - Use double integrals to find the radius of gyration of a triangular lamina
 - Use double integrals to find the radius of gyration of a trapezoidal lamina
 - Use double integrals and technology to find the radius of gyration of a circular lamina
 - Use double integrals and technology to find the radius of gyration of an elliptical lamina
- Center of Mass in Three Dimensions
 - Use triple integrals to find the mass and center of mass of a solid
 - Use triple integrals to find the mass of a solid
 - Use triple integrals to find the moments of a solid
 - Use triple integrals to find the center of mass of a solid
 - Use triple integrals to find the centroid of a solid
- Moments of Inertia in Three Dimensions
 - Use triple integrals to find the moments of inertia of a solid
 - Use triple integrals to find the moments of inertia of a solid

15.7 Change of Variables in Multiple Integrals

- Transformations of Variables
-

- Understand the concept of a transformation of variables
 - Determine the image of a point under a given transformation of variables in two dimensions
 - Determine the image of a region under a given transformation of variables in two dimensions
 - Determine the image of a region under a given transformation of variables in three dimensions
 - Sketch the image of a region under a given transformation of variables in two dimensions
 - Determine a transformation that maps one region to another region in two dimensions
 - One-to-one and Inverse Transformations
 - Understand one-to-one and inverse transformations
 - Determine if a transformation in two dimensions is one-to-one
 - Determine if a transformation in three dimensions is one-to-one
 - Determine an inverse transformation for a given one-to-one transformation containing a linear function in two dimensions
 - Determine an inverse transformation for a given one-to-one transformation containing a linear function in three dimensions
 - Determine an inverse transformation for a given one-to-one transformation containing a transcendental function in two dimensions
 - Determine the preimage of a region under a given one-to-one transformation in two dimensions
 - Determine the preimage of a region under a given one-to-one transformation in three dimensions
 - Determine the image of a region using the inverse of a given one-to-one transformation
 - Sketch the image of a region using the inverse of a given one-to-one transformation of variables in two dimensions
 - Use an inverse transformation to determine a transformation that maps one region to another region in two dimensions
 - Jacobians for Functions of Two Variables
 - Find the Jacobian for a transformation of a function of two variables
 - Find the Jacobian for a transformation of an algebraic function of two variables
 - Find the Jacobian for a transformation of an exponential function of two variables
 - Find the Jacobian for a transformation of a trigonometric function of two variables
 - Change of Variables for Double Integrals
 - Set up a double integral of a function using a change of variables
 - Set up a double integral of a polynomial function using a change of variables with a given transformation
 - Set up a double integral of a radical function using a change of variables with a given transformation
-

- Set up a double integral of a trigonometric function using a change of variables with a given transformation
 - Set up a double integral of an exponential function using a change of variables with a given transformation
 - Evaluate a double integral using a change of variables
 - Evaluate a double integral of a polynomial function using a change of variables with a given transformation
 - Evaluate a double integral of a radical function using a change of variables with a given transformation
 - Evaluate a double integral of a trigonometric function using a change of variables with a given transformation
 - Evaluate a double integral of an exponential function using a change of variables with a given transformation
 - Evaluate a double integral of a function using an appropriate change of variables
 - Jacobians for Functions of Three Variables
 - Find the Jacobian for a transformation of a function of three variables
 - Find the Jacobian for a transformation of an algebraic function of three variables
 - Find the Jacobian for a transformation of a trigonometric function of three variables
 - Change of Variables for Triple Integrals
 - Set up a triple integral of a function using a change of variables
 - Setup a triple integral of a polynomial function using a change of variables with a given transformation
 - Setup a triple integral of a radical function using a change of variables with a given transformation
 - Setup a triple integral of an exponential function using a change of variables with a given transformation
 - Evaluate a triple integral using a change of variables
 - Evaluate a triple integral of a polynomial function using a change of variables with a given transformation
 - Evaluate a triple integral of a radical function using a change of variables with a given transformation
 - Evaluate a triple integral of an exponential function using a change of variables with a given transformation
 - Finding Area and Volume Using a Change of Variables
 - Find area using a double integral with a change of variables
 - Set up a double integral using a change of variables to find the area of a region
 - Evaluate a double integral using a change of variables to find the area of a region
 - Find volume using a change of variables
 - Evaluate a double integral using a change of variables to find the volume of a solid
 - Evaluate a triple integral using a change of variables to find the volume of a solid
-

Chapter 16: Vector Calculus

16.1 Vector Fields

- Properties of Vector Fields
 - Find the vector associated with a given point in a vector field in two dimensions
 - Find the vector associated with a given point in a vector field defined by polynomial functions of two variables
 - Find the vector associated with a given point in a vector field defined by rational functions of two variables
 - Find the vector associated with a given point in a vector field defined by radical functions of two variables
 - Find the vector associated with a given point in a vector field defined by trigonometric functions of two variables
 - Find the vector associated with a given point in a vector field defined by exponential functions of two variables
 - Find the vector associated with a given point in a vector field in three dimensions
 - Find the vector associated with a given point in a vector field defined by polynomial functions of three variables
 - Find the vector associated with a given point in a vector field defined by rational functions of three variables
 - Find the vector associated with a given point in a vector field defined by radical functions of three variables
 - Find the vector associated with a given point in a vector field defined by trigonometric functions of three variables
 - Find the vector associated with a given point in a vector field defined by exponential functions of three variables
 - Use properties of vector fields
 - Determine whether a vector field is radial, rotational, or neither
 - Determine whether a vector field is a unit vector field
 - Draw A Vector Field
 - Sketch a vector field from a given equation in two dimensions
 - Identify the plot of a two-dimensional vector field
 - Sketch a two-dimensional radial vector field
 - Sketch a two-dimensional rotational vector field
 - Sketch a two-dimensional vector field defined by polynomial functions
 - Sketch a two-dimensional vector field defined by rational functions
 - Sketch a two-dimensional vector field defined by trigonometric functions
 - Sketch a vector field from a given equation in three dimensions
 - Identify the plot of a three-dimensional vector field
 - Sketch a three-dimensional vector field defined by polynomial functions
 - Gradient Fields
 - Find the gradient vector field for a given function of two variables
 - Find the gradient vector field of an algebraic function of two variables
 - Find the gradient vector field of a transcendental function of two variables
 - Sketch the gradient vector field for a given function in two dimensions
 - Identify the plot of a gradient vector field in two dimensions
-

- Sketch the plot of a gradient vector field of an algebraic function in two dimensions
- Sketch the plot of a gradient vector field of a transcendental function in two dimensions
- Find the gradient vector field for a given function of three variables
 - Find the gradient vector field of an algebraic function of three variables
 - Find the gradient vector field of a transcendental function of three variables
- Determine if a vector field is not conservative
 - Use the cross-partial property to determine if a vector field defined by algebraic functions in two dimensions is not conservative
 - Use the cross-partial property to determine if a vector field defined by transcendental functions in two dimensions is not conservative
 - Use the cross-partial property to determine if a vector field defined by algebraic functions in three dimensions is not conservative
 - Use the cross-partial property to determine if a vector field defined by transcendental functions in three dimensions is not conservative

16.2 Line Integrals

- **Scalar Line Integrals**
 - Convert a line integral to an ordinary integral
 - Find a parameterization of a line segment
 - Find a parameterization of a quarter-circle
 - Convert a line integral of a polynomial function to an ordinary integral
 - Convert a line integral over a line segment to an ordinary integral
 - Evaluate a scalar line integral
 - Evaluate a scalar line integral of a polynomial function over a parameterized line segment
 - Evaluate scalar line integral of a trigonometric function over a parameterized trigonometric curve
 - Evaluate a scalar line integral over a parameterized trigonometric curve in three dimensions
 - Evaluate a scalar line integral of a polynomial function over a parabola
 - Evaluate a scalar line integral of a polynomial function over a half circle
 - Use a line integral to find an arclength
 - Calculate arc length of a parameterized trigonometric curve
 - Calculate arc length of a parameterized polynomial curve in two and three dimensions
 - **Vector Line Integrals**
 - Calculate a vector line integral
 - Calculate a vector line integral of a polynomial vector field along a parameterized polynomial curve
 - Calculate a vector line integral of a rational function along a parameterized polynomial curve
 - Set up a vector line integral written in expanded form over a parameterized trigonometric curve in three dimensions.
-

- Calculate a vector line integral written in expanded form over a parameterized polynomial curve
- Use properties to compute a vector line integral
 - Use properties to compute a vector line integral of a vector field over a rectangle
 - Use properties to compute an equivalent vector line integral
- Flux and Circulation
 - Compute a Flux of a vector field
 - Compute the flux of a polynomial vector field across a line segment
 - Compute the flux of a vector field across a parameterized curve
 - Compute the flux of a vector field across a circle
 - Calculate a Circulation of a vector field
 - Calculate the circulation of a polynomial vector field F moving counterclockwise along a circle
 - Calculate the circulation of a rational vector field F along a circle
- Applications of Line Integrals
 - Calculate mass of a wire
 - Calculate the mass of a wire that follows a circle in two dimensions
 - Calculate the mass of a wire that follows a parameterized polynomial curve in three dimensions
 - Calculate the mass of a wire that follows a parameterized trigonometric curve in three dimensions
 - Find work done by a force field
 - Find the work done by a force field F on a particle moving along a three-dimensional path
 - Find the work done by a force field F on a particle moving along a line segment in three dimensions
 - Use properties of vector line integrals to describe work done along a path

16.3 Conservative Vector Fields

- Curves and Regions
 - Describe simple and closed curves
 - Identify simple and closed curves involving polynomial parameterizations
 - Identify simple and closed curves involving trigonometric parameterizations
 - Identify simple and closed curves involving a combination of polynomial and trigonometric parameterizations
 - Define connected and simply connected regions
 - Identify simply connected regions involving trigonometric curves
 - Identify simply connected regions involving polar curves
 - Identify simply connected regions involving circles
 - The Fundamental Theorem for Line Integrals
 - Use the Fundamental Theorem for Line Integrals to evaluate a line integral in a vector field given a potential function
 - Evaluate a line integral in two dimensions using the Fundamental Theorem of Line Integrals involving an algebraic vector field
 - Evaluate a line integral in three dimensions using the Fundamental Theorem of Line Integrals involving an algebraic vector field
-

- Evaluate a line integral in two dimensions using the Fundamental Theorem of Line Integrals involving a transcendental vector field
- Evaluate a line integral in three dimensions using the Fundamental Theorem of Line Integrals involving a transcendental vector field
- Determine if a vector field is conservative on a given region
 - Determine if a vector field in two dimensions involving algebraic functions is conservative on a given region
 - Determine if a vector field in three dimensions involving algebraic functions is conservative on a given region
 - Determine if a vector field in two dimensions involving transcendental functions is conservative on a given region
 - Determine if a vector field in three dimensions involving transcendental functions is conservative on a given region
- Find a potential function for a simply connected vector field
 - Find a potential function for an algebraic vector field in two dimensions
 - Find a potential function for an algebraic vector field in three dimensions
 - Find a potential function for a transcendental vector field in two dimensions
 - Find a potential function for a transcendental vector field in three dimensions
- Use the Fundamental Theorem of Line Integrals to evaluate a line integral
 - Given an algebraic vector field in two dimensions, find the potential function and use it to compute a line integral
 - Given an algebraic vector field in three dimensions, find the potential function and use it to compute a line integral
 - Given a transcendental vector field in two dimensions, find the potential function and use it to compute a line integral
 - Given a transcendental vector field in three dimensions, find the potential function and use it to compute a line integral

16.4 Green's Theorem

- The Circulation Form of Green's Theorem
 - Use the circulation form of Green's Theorem to evaluate line integrals
 - Verify the conditions of Green's Theorem
 - Use Green's Theorem to evaluate a line integral along a closed rectangular path through a vector field involving algebraic functions
 - Use Green's Theorem to evaluate a line integral along a closed rectangular path through a vector field involving transcendental functions
 - Use Green's Theorem to evaluate a line integral along a closed circular path through a vector field involving algebraic functions
 - Use Green's Theorem to evaluate a line integral along a closed circular path through a vector field involving transcendental functions
 - Use Green's Theorem to evaluate a line integral along a closed triangular path through a vector field involving algebraic functions
 - Use Green's Theorem to evaluate a line integral along a closed triangular path through a vector field involving transcendental functions
 - Use Green's Theorem to evaluate a line integral along a closed elliptical path
 - Use Green's Theorem to evaluate a line integral along a general closed path
-

- The Flux Form of Green's Theorem
 - Use the flux form of Green's Theorem
 - Use Green's Theorem to find the flux across a rectangle through a vector field involving algebraic functions
 - Use Green's Theorem to find the flux across a rectangle through a vector field involving transcendental functions
 - Use Green's Theorem to find the flux across a circle through a vector field involving algebraic functions
 - Use Green's Theorem to find the flux across a circle through a vector field involving transcendental functions
 - Use Green's Theorem to find the flux across a triangle through a vector field involving algebraic functions
 - Use Green's Theorem to find the flux across a triangle through a vector field involving transcendental functions
 - Use Green's Theorem to find the flux across an ellipse
 - Source Free Vector Fields
 - Understand source free vector fields
 - Determine if a vector field is source free
 - Find a stream function for a source free vector field involving polynomial functions
 - Find a stream function for a source free vector field involving transcendental functions
 - Determine if a polynomial function satisfies Laplace's equation
 - Determine if a transcendental function satisfies Laplace's equation
 - Find a potential function that satisfies Laplace's equation for a source free and conservative vector field
 - Green's Theorem on General Regions
 - Use Green's Theorem on general regions
 - Use Green's Theorem to evaluate a line integral on a region with holes through a vector field involving algebraic functions
 - Use Green's Theorem to evaluate a line integral on a region with holes through a vector field involving transcendental functions
 - Applications of Green's Theorem
 - Use Green's Theorem to solve applications
 - Use Green's Theorem to calculate work along a closed rectangular path through a vector field involving algebraic functions
 - Use Green's Theorem to calculate work along a closed rectangular path through a vector field involving transcendental functions
 - Use Green's Theorem to calculate work along a closed circular path through a vector field involving algebraic functions
 - Use Green's Theorem to calculate work along a closed circular path through a vector field involving transcendental functions
 - Use Green's Theorem to calculate work along a closed triangular path through a vector field involving algebraic functions
-

- Use Green's Theorem to calculate work along a closed triangular path through a vector field involving transcendental functions
- Use Green's Theorem to calculate work along a closed elliptical path
- Use Green's Theorem to calculate work along a general closed path through a vector field involving algebraic functions
- Use Green's Theorem to calculate work along a general closed path through a vector field involving transcendental functions
- Use Green's Theorem to find the area of a region enclosed by a curve
- Use Green's Theorem to find the flux of a fluid through a region

16.5 Divergence and Curl

- Divergence
 - Find the divergence of a vector field
 - Find the divergence of a two-dimensional vector field involving polynomial functions
 - Find the divergence of a two-dimensional vector field involving radical functions
 - Find the divergence of a two-dimensional vector field involving trigonometric functions
 - Find the divergence of a three-dimensional vector field involving polynomial functions
 - Find the divergence of a three-dimensional vector field involving trigonometric functions
 - Find the divergence of a three-dimensional vector field involving exponential functions
 - Find the divergence of a field at a point
 - Find the divergence of a two-dimensional vector field involving polynomial functions at a point
 - Find the divergence of a two-dimensional vector field involving radical functions at a point
 - Find the divergence of a two-dimensional vector field involving trigonometric functions at a point
 - Find the divergence of a three-dimensional vector field involving polynomial functions at a point
 - Find the divergence of a three-dimensional vector field involving trigonometric functions at a point
 - Find the divergence of a three-dimensional vector field involving exponential functions at a point
 - Use divergence to determine whether a vector field is source free
 - Determine whether a vector field involving polynomials is source-free
 - Determine whether a vector field involving trigonometric functions is source-free
 - Curl
 - Find the curl of a vector field
 - Find the curl of a three-dimensional vector field involving polynomial functions
 - Find the curl of a three-dimensional vector field involving trigonometric functions
 - Find the curl of a three-dimensional vector field involving exponential functions
 - Find the curl of a two-dimensional vector field involving polynomial functions
-

- Find the curl of a two-dimensional vector field involving trigonometric functions
 - Find the curl of a two-dimensional vector field involving exponential functions
 - Find the curl of a vector field at a point
 - Find the curl of a three-dimensional vector field involving polynomial functions at a point
 - Find the curl of a three-dimensional vector field involving trigonometric functions at a point
 - Find the curl of a three-dimensional vector field involving exponential functions at a point
 - Find the curl of a two-dimensional vector field involving polynomial functions at a point
 - Find the curl of a two-dimensional vector field involving trigonometric functions at a point
 - Find the curl of a two-dimensional vector field involving exponential functions at a point
 - Applications of Curl
 - Use the curl to determine whether a vector field is conservative
 - Determine whether a two-dimensional vector field involving polynomials is conservative
 - Determine whether a two-dimensional vector field involving trigonometric functions is conservative
 - Determine whether a three-dimensional vector field involving polynomials is conservative
 - Determine whether a three-dimensional vector field involving trigonometric functions is conservative
 - Determine whether a vector field could be the curl of another vector field
 - Determine whether a two-dimensional vector field involving polynomials could be the curl of another vector field
 - Determine whether a two-dimensional vector field involving trigonometric functions could be the curl of another vector field
 - Determine whether a three-dimensional vector field involving polynomials could be the curl of another vector field
 - Use the Laplace operator to determine whether a scalar function is harmonic
 - Determine whether a polynomial function of two variables is harmonic
 - Determine whether a transcendental function of two variables is harmonic
 - Determine whether a polynomial function of three variables is harmonic
 - Determine whether a transcendental function of three variables is harmonic
 - Divergence and Flow
 - Use divergence to understand the flow of a fluid
 - Use divergence to find the flow of a fluid with a polynomial velocity field at a point in two dimensions
 - Use divergence to find the flow of a fluid with a trigonometric velocity field at a point in two dimensions
 - Use divergence to find the flow of a fluid with a polynomial velocity field at a point in three dimensions
-

- Use divergence to find the flow of a fluid with a trigonometric velocity field at a point in three dimensions
- Find a point at which the inflow and outflow of a fluid are equal in two dimensions
- Use divergence to understand electromagnetic fields
 - Determine whether a two-dimensional vector field involving polynomials could be a magnetic field
 - Determine whether a two-dimensional vector field involving trigonometric functions could be a magnetic field
 - Determine whether a three-dimensional vector field could be a magnetic field
 - Determine whether a polynomial function in two variables could be the potential function of an electrostatic field
 - Determine whether a transcendental function in two variables could be the potential function of an electrostatic field
 - Determine whether a polynomial function in three variables could be the potential function of an electrostatic field
 - Determine whether a transcendental function in three variables could be the potential function of an electrostatic field

16.6 Surface Integrals

- Parametric Surfaces
 - Find a parameterization for a surface
 - Identify the surface described by a parameterization
 - Find a parameterization for a surface
 - Identify a smooth surface
 - Find the surface area of a parameterized surface
 - Set up a surface integral to find a surface area given a polynomial parameterization
 - Set up a surface integral to find a surface area given a trigonometric parameterization
 - Find a surface area given a polynomial parameterization
 - Find a surface area given a trigonometric parameterization
 - Find the surface area of a planar surface
 - Find the surface area of a cylindrical surface
 - Find the surface area of a spherical surface
 - Find the surface area of a surface of revolution
 - Set up a surface integral to find the surface area of a surface of revolution
 - Find the surface area of a surface of revolution
 - Scalar Surface Integrals
 - Evaluate a surface integral of a scalar valued function
 - Evaluate a surface integral of a scalar valued function given a polynomial parameterization
 - Evaluate a surface integral of a scalar valued function given a trigonometric parameterization
 - Evaluate the surface integral of a scalar valued function over a planar surface
 - Evaluate the surface integral of a scalar valued function over a cylindrical surface
-

- Surface Integrals of a Vector Field
 - Understand the orientation of a surface
 - Find an orientation for a parametric surface
 - Find a tangent plane to a parametric surface
 - Evaluate a surface integral of a vector field
 - Evaluate a surface integral of a vector field given a polynomial parameterization
 - Evaluate a surface integral of a vector field given a trigonometric parameterization
 - Evaluate the surface integral of a vector field over a planar surface
 - Evaluate the surface integral of a vector field over a spherical surface
- Applications of Surface Integrals
 - Use Surface integrals to Solve Applications
 - Find the mass of a sheet described by a parameterized surface
 - Find a mass flow rate using a surface integral
 - Find a heat flow rate using a surface integral

16.7 Stokes' Theorem

- Understand and Apply Stokes' Theorem
 - Understand Stokes' theorem
 - Write a line integral that is equivalent to a given surface integral over a parabolic surface with circular boundary
 - Write a line integral that is equivalent to a given surface integral over a general surface with triangular boundary
 - Write a surface integral that is equivalent to a given line integral along a triangular path
 - Write a surface integral that is equivalent to a given line integral along an elliptic path
 - Use Stokes' theorem to evaluate a surface integral
 - Use Stokes' theorem to evaluate a surface integral over a planar surface with triangular boundary
 - Use Stokes' theorem to evaluate a surface integral over a planar surface with elliptic boundary
 - Use Stokes' theorem to evaluate a surface integral over a parabolic surface with semicircular boundary
 - Use Stokes' theorem to evaluate a surface integral over a hemispherical surface with circular boundary
 - Use Stokes' theorem to evaluate a surface integral over an elliptic surface with elliptic boundary
 - Use Stokes' theorem to evaluate a line integral
 - Use Stokes' theorem to evaluate a line integral along a triangular path
 - Use Stokes' theorem to evaluate a line integral along a circular path
 - Use Stokes' theorem to evaluate a line integral along an elliptic path
 - Use Stokes' theorem in applications
 - Use Stokes' theorem to find the circulation of a fluid along a triangular path
 - Use Stokes' theorem to find the circulation of a fluid along a circular path
-

- Use Faraday's law to compute the curl of an electric field given a constant magnetic field
- Use Faraday's law to compute the curl of an electric field given a uniform magnetic field that varies with time
- Use Faraday's law to compute the curl of an electric field given a radial magnetic field that varies with time

16.8 The Divergence Theorem

- Understand and Apply the Divergence Theorem
 - Understand the divergence theorem
 - Write a triple integral that is equivalent to a given flux integral across a closed rectangular surface
 - Write a triple integral that is equivalent to a given flux integral across a closed polyhedral surface
 - Write a triple integral that is equivalent to a given flux integral across a closed cylindrical surface
 - Write a triple integral that is equivalent to a given flux integral across a closed parabolic surface
 - Write a triple integral that is equivalent to a given flux integral across a closed conical surface
 - Write a triple integral that is equivalent to a given flux integral across a closed spherical surface
 - Use the divergence theorem to determine the flux of a vector field across a closed surface
 - Use the divergence theorem to determine the flux of an algebraic vector field across a closed rectangular surface
 - Use the divergence theorem to determine the flux of a transcendental vector field across a closed rectangular surface
 - Use the divergence theorem to determine the flux of an algebraic vector field across a closed polyhedral surface
 - Use the divergence theorem to determine the flux of an algebraic vector field across a closed cylindrical surface
 - Use the divergence theorem to determine the flux of a transcendental vector field across a closed cylindrical surface
 - Use the divergence theorem to determine the flux of an algebraic vector field across a closed parabolic surface
 - Use the divergence theorem to determine the flux of a transcendental vector field across a closed parabolic surface
 - Use the divergence theorem to determine the flux of an algebraic vector field across a closed conical surface
 - Use the divergence theorem to determine the flux of an algebraic vector field across a closed spherical surface
 - Use the divergence theorem to determine the flux of a transcendental vector field across a closed spherical surface
 - Use the divergence theorem in applications

- Use the divergence theorem to determine the flux of a fluid with a polynomial velocity field across a closed surface
- Use the divergence theorem to determine the flux of a fluid with an exponential velocity field across a closed surface
- Use the divergence theorem to determine the flux of an electrostatic field across a closed cylindrical surface
- Use the divergence theorem to determine the flux of an electrostatic field across a closed spherical surface
- Use the divergence theorem to determine the heat flux of a polynomial temperature distribution across a closed surface
- Use the divergence theorem to determine the heat flux of an exponential temperature distribution across a closed surface

Chapter 17: Second-Order Differential Equations

17.1 Second-Order Linear Equations

- Understanding and Classifying Linear Differential Equations
 - Understand and classify linear differential equations
 - Classify a differential equation as linear or nonlinear
 - Classify a linear differential equation as homogeneous or nonhomogeneous
 - Verify a solution to a differential equation
 - The Superposition Principle
 - Apply the superposition principle to second-order homogeneous linear differential equations
 - Verify the superposition principle for homogeneous differential equations
 - Determine if a given pair of functions is linearly dependent or linearly independent
 - Find the general solution to a homogeneous differential equation given two linearly independent solutions
 - Solving Homogeneous Second-Order Linear Differential Equations
 - Solve homogeneous second-order linear differential equations with constant coefficients
 - Determine the characteristic equation of a second-order linear differential equation
 - Determine the type and number of roots of the characteristic equation of a second-order linear differential equation
 - Solve a second-order linear differential equation for which the characteristic equation has two distinct real roots
 - Solve a second-order linear differential equation for which the characteristic equation has a repeated real root
 - Solve a second-order linear differential equation for which the characteristic equation has complex roots
 - Determine the second-order linear homogeneous differential equation with constant coefficients that has a given general solution
 - Solve a second-order linear differential equation
 - Initial Value Problems for Second-Order Linear Differential Equations
 - Solve initial-value and boundary-value problems involving linear differential equations
-

- Solve an initial-value problem for a second-order differential equation
- Solve a second-order differential equation representing a spring problem
- Solve a boundary-value problem for a second-order differential equation

17.2 Nonhomogeneous Linear Equations

- Finding Solutions Using Undetermined Coefficients
 - Write the general solution to a nonhomogeneous differential equation
 - Given a particular solution that is a polynomial function, find the general solution to a nonhomogeneous differential equation
 - Given a particular solution that is an exponential function, find the general solution to a nonhomogeneous differential equation
 - Given a particular solution that is a trigonometric function, find the general solution to a nonhomogeneous differential equation
 - Given a particular solution that is a combination of polynomial, exponential, or trigonometric functions, find the general solution to a nonhomogeneous differential equation
 - Write the form for the particular solution for the method of undetermined coefficients
 - Given a nonhomogeneous differential equation, determine the form for the particular solution for the method of undetermined coefficients
 - Solve a nonhomogeneous differential equation using the method of undetermined coefficients
 - Use the method of undetermined coefficients to solve a nonhomogeneous differential equation when $r(x)$ is a polynomial function
 - Use the method of undetermined coefficients to solve a nonhomogeneous differential equation when $r(x)$ is an exponential function
 - Use the method of undetermined coefficients to solve a nonhomogeneous differential equation when $r(x)$ is a trigonometric function
 - Use the method of undetermined coefficients to solve a nonhomogeneous differential equation when $r(x)$ is a combination of polynomial, exponential or trigonometric functions
 - Use the method of undetermined coefficients to solve a nonhomogeneous differential equation when $r(x)$ is a solution of the complementary equation
 - Initial Value Problems for Nonhomogeneous Differential Equations
 - Solve an initial value problem for a nonhomogeneous differential equation
 - Given a particular solution, determine the unique solution satisfying an initial value problem for a nonhomogeneous differential equation.
 - Use the method of undetermined coefficients to solve an initial value problem when $r(x)$ is a trigonometric function
 - Use the method of undetermined coefficients to solve an initial value problem when $r(x)$ is combination of polynomial and exponential functions
 - Finding Solutions Using Variation of Parameters
 - Find a particular solution for a nonhomogeneous differential equation using the method of variation of parameters
 - Use Cramer's rule to solve a system of equations with variable coefficients
 - Use the method of variation of parameters to find a particular solution to a nonhomogeneous differential equation when $r(x)$ is a polynomial function
-

- Use the method of variation of parameters to find a particular solution to a nonhomogeneous differential equation when $r(x)$ is an exponential function
- Use the method of variation of parameters to find a particular solution to a nonhomogeneous differential equation when $r(x)$ is a trigonometric function
- Use the method of variation of parameters to find a particular solution to a nonhomogeneous differential equation when $r(x)$ is a combination of polynomial and exponential functions
- Use the method of variation of parameters to find a particular solution to a nonhomogeneous differential equation with nonconstant coefficients
- Find a general solution for a nonhomogeneous differential equation using the method of variation of parameters
 - Use the method of variation of parameters to find a general solution to a nonhomogeneous differential equation when $r(x)$ is a polynomial function
 - Use the method of variation of parameters to find a general solution to a nonhomogeneous differential equation when $r(x)$ is an exponential function
 - Use the method of variation of parameters to find a general solution to a nonhomogeneous differential equation when $r(x)$ is a trigonometric function
 - Use the method of variation of parameters to find a general solution to a nonhomogeneous differential equation when $r(x)$ is a combination of polynomial, exponential or trigonometric functions

17.3 Applications

- Simple Harmonic Motion
 - Solve second-order differential equations involving simple harmonic motion
 - Solve second-order differential equations involving simple harmonic motion
Solve a second-order differential equation representing damped simple harmonic motion
 - Solve second-order differential equations involving simple harmonic motion
Express a sum of trigonometric functions as a single trigonometric function
 - Damped Spring-Mass Systems
 - Solve second-order differential equations involving damped simple harmonic motion
 - Solve a second-order differential equation representing an overdamped spring-mass system
 - Solve a second-order differential equation representing a critically damped spring-mass system
 - Solve a second-order differential equation representing an underdamped spring-mass system
 - Solve second-order differential equations involving forced simple harmonic motion
 - Find the steady-state solution to a second-order differential equation representing forced simple harmonic motion
 - Find the transient solution to a second-order differential equation representing forced simple harmonic motion
 - RLC Series Circuits
 - Solve second-order differential equations involving an RLC circuit
-

- Find the charge on the capacitor in an RLC series circuit represented by a second-order differential equation
- Find the current in an RLC series circuit represented by a second-order differential equation

17.4 Series Solutions of Differential Equations

- Solve Differential Equations with Power Series
 - Use power series to solve a first-order differential equation
 - Find a power series solution to a first-order differential equation with constant coefficients
 - Find a power series solution to a first-order differential equation with nonconstant coefficients
 - Use power series to solve a second-order differential equation
 - Find a power series solution to a second-order differential equation with constant coefficients
 - Find a power series solution to a second-order differential equation with nonconstant coefficients
 - Find a power series solution to a second-order initial-value problem with constant coefficients
 - Find a power series solution to a second-order initial-value problem with nonconstant coefficients

Precalculus Review

Chapter 1: Precalculus Prerequisites

1.1 Algebra Essentials

- Sets and Venn Diagrams
 - Represent a set using a written description and the roster method
 - Identify subsets, universal sets, and empty sets
 - Illustrate two sets using Venn diagram and set notation
 - Set Operations
 - Determine the complement of a set using Venn diagrams or set notation
 - Determine the intersection of two sets using Venn diagrams or set notation
 - Determine the union of two sets using Venn diagrams or set notation
 - Perform operations on sets
 - Properties of Real Numbers and Order of Operations
 - Distinguish between natural numbers, whole numbers, and integers
 - Distinguish between rational and irrational numbers
 - Perform calculations using order of operations
 - Use the inverse and identity properties of real numbers
 - Use the commutative, associative, and distributive properties of real numbers
 - Evaluate and Simplify Algebraic Expressions
 - Evaluate algebraic expressions with a single variable
 - Evaluate algebraic expressions with two variables
 - Identify constants and variables
 - Use a formula
-

- Simplify algebraic expressions
- Evaluate and Simplify Absolute Value Expressions
 - Understand absolute value and evaluate absolute value expressions with the order of operations
 - Evaluate algebraic expressions involving absolute value

1.2 Exponents and Scientific Notation

- Product, Quotient, and Power Properties of Exponents
 - Understand exponent notation
 - Use the product rule of exponents
 - Use the quotient rule of exponents
 - Use the power rule of exponents
- Negative Exponents and Simplifying Exponential Expressions
 - Use the negative and zero exponent rule
 - Find the power of a product
 - Find the power of a quotient
 - Simplify exponential expressions
- Scientific Notation
 - Convert between standard and scientific notation
 - Multiply and divide numbers in scientific notation

1.3 Radicals and Rational Exponents

- Simplify Radicals
 - Evaluate square roots
 - Use the product rule to simplify square roots
 - Use the quotient rule to simplify square roots
- Operations with Radicals
 - Add and subtract square roots
 - Rationalize denominators with a monomial denominator
 - Rationalize denominators using the conjugate
- Rational Exponents and Higher Order Radicals
 - Simplify nth roots
 - Operations with nth roots
 - Switch between radical and rational exponent form
 - Evaluate expressions with rational exponents

1.4 Factoring Polynomials

- Factor Quadratics
 - Factor the greatest common factor of a polynomial
 - Factor a trinomial
 - Factor a trinomial by grouping
 - Factor Quadratics with Special Products
 - Factor a perfect square trinomial
 - Factor a difference of squares
 - Factor Cubics
 - Factor a cubic by grouping
 - Factor the sum and difference of cubes
 - Factor Expressions with Fractional or Negative Exponents
-

- Factor expressions using fractional or negative exponents
- Factor expressions using greatest common factor and other technique

1.5 Rational Expressions

- Multiply and Divide Rational Expressions
 - Simplify rational expressions
 - Multiply rational expressions
 - Divide rational expressions
- Add and Subtract Rational Expressions and Simplify Complex Rational Expressions
 - Add and subtract rational expressions
 - Simplify complex rational expressions

Chapter 2: Equations and Inequalities

2.1 Linear and Rational Equations in One Variable

- Solve Linear Equations in One Variable
 - Identify identity, conditional, and inconsistent equations
 - Solve equations in one variable algebraically, variable just on one side
 - Solve equations in one variable algebraically, variable on both sides
- Solve Rational Equations
 - Solve a rational equation, monomials in denominator
 - Solve a rational equation, binomials in denominator
 - Solve a rational equation, requires factoring to find least common denominator
- Identify Slopes and Intercepts
 - Find the slope of a line given two points
 - Understand the relationship between the slope and y-intercept of a line and its equation
 - Find x -intercepts and y -intercepts
- Find Linear Equations
 - Find equation of a line, in slope-intercept form, given slope and one point (point-slope formula)
 - Find equation, in slope-intercept form, of a line passing through two given points
 - Given slope and intercept, find the equation of a line and write it in standard form
 - Find the equation of vertical and horizontal lines
- Parallel and Perpendicular Lines
 - Given the equations of two lines, determine whether their graphs are parallel or perpendicular
 - Write the equation of a line parallel to a given line
 - Write the equation of a line perpendicular to a given line

2.2 Models and Applications

- Word Problems with Linear Equations
 - Set up a linear equation to solve a real-world application
 - Translate verbal expressions into mathematical expressions
 - Use a formula to solve a real-world application
 - Problem Solving
 - Solve simple interest applications
 - Solve a formula for a specified variable
 - Use a formula to solve a geometric application
-

- Use the Pythagorean theorem

2.3 Quadratic Equations

- Solve Quadratic Equations by Factoring
 - Solve quadratic equations by factoring, leading coefficient 1
 - Solve quadratic equations by factoring, leading coefficient > 1
- Complete the Square
 - Solve quadratic equations by the square root property
 - Solve quadratic equations by completing the square
- Quadratic Formula
 - Use the discriminant to classify the solutions of a quadratic equation
 - Solve quadratic equations by using the quadratic formula

2.4 Other Types of Equations

- Solve Higher Order Equations with Factoring
 - Solve equations by factoring out the greatest common factor
 - Solve equations by factoring with grouping
- Solve Equations Quadratic in Form by Factoring
 - Solve fourth-degree equation in quadratic form
 - Solve quadratic with binomial
- Solve Radical Equations
 - Solve radical equations with a single radical
 - Solve radical equations with two radicals
- Solve Other Types of Equations
 - Solve equations using reciprocal exponents
 - Solve equations involving rational exponents by factoring out the greatest common factor
 - Solve rational equation which leads to a quadratic

2.5 Linear Inequalities and Absolute Value Inequalities

- Interval Notation and Inequalities
 - Use interval notation
 - Use properties of inequalities
 - Solve simple inequalities in one variable algebraically
 - Solve compound inequalities in one variable algebraically
- Absolute Value Equations and Inequalities
 - Solve absolute value equations
 - Solve absolute value inequalities
- Applications with Linear Inequalities
 - Translate words to an inequality and solve applications with linear inequalities
 - Solve applications with compound inequalities
 - Solve applications with absolute value

2.6 Inequalities Requiring Factoring

- Rational and Quadratic Inequalities
 - Solve quadratic inequalities in one variable, graph the solution set, and express the solution set using interval notation
 - Solve inequalities that involve rational expressions, graph the solution sets, and express the solution set using interval notation
-

Chapter 3: Functions

3.1 Functions and Function Notation

- Relations and Functions
 - Identify domain and range from a set of ordered pairs
 - Determine whether a relation represents a function
 - Use the vertical line test to identify functions
- One-to-One Functions
 - Determine whether a function is one-to-one
 - Use the horizontal line test to identify one-to-one functions
- Function Notation
 - Evaluate a function using function notation
 - Solve a function using function notation
 - Evaluate or solve a function from a table
 - Evaluate or solve a function from a graph

3.2 Absolute Value Functions and Other Toolkit Functions

- Graph Absolute Value Functions
 - Graph an absolute value function
- Toolkit Functions
 - Identify graphs of toolkit functions

3.3 Domain and Range

- Domain and Range of Functions
 - Find the domain of a function defined by an equation
 - Find the domain and range of a function defined by a graph
- Piecewise Functions
 - Graph piecewise-defined functions
 - Evaluate piecewise-defined functions

3.4 Rates of Change and Behavior of Graphs

- Graphical Properties of Functions
 - Use a graph to determine intervals of increase and decrease
 - Use a graph to locate the absolute maximum and absolute minimum
 - Find the average rate of change of a function
 - Use a graph to determine local extrema
- Difference Quotients
 - Determine the difference quotient

3.5 Composition of Functions

- Combinations of Functions
 - Combine functions using algebraic operations
 - Create a new function by composition of functions
 - Evaluate Composite Functions
 - Evaluate composite functions given a table of values
 - Evaluate composite functions given the graph of functions
 - Evaluate composite functions given explicit functions
 - Properties of Composite Functions
 - Find the domain of a composite function
-

- Decompose a composite function into its component functions

3.6 Inverse Functions

- Inverse Function Values
 - Verify inverse function ordered pairs
 - Given graph of a function, find value of inverse function
 - Given table of values of a function, find value of inverse function
- Find Inverse Functions
 - Verify inverse function pairs algebraically
 - Determine the domain and range of an inverse function, and restrict the domain of a function to make it one-to-one
 - Given function, find the inverse function
 - Use the graph of a one-to-one function to graph its inverse function on the same axes

3.7 Circles

- Graphs of Circles
 - Given the equation of a circle not in standard form, determine the standard form by completing the square
 - Determine the center and radius of a circle from the standard equation of a circle and sketch its graph

Chapter 4: Linear Functions and Modeling

4.1 Linear Functions

- Interpretations of Linear Functions
 - Represent a linear function in table form
 - Determine whether a linear function is increasing, decreasing, or constant
 - Interpret slope as a rate of change
 - Represent a real-world application as a linear function
 - Graph linear functions

Chapter 5: Polynomial and Rational Functions

5.1 Quadratic Functions

- Characteristics of Parabolas
 - Determine axis of symmetry and vertex of parabolas from a graph
 - Determine x- and y-intercepts of parabolas from a graph
 - Find the direction a parabola opens and its axis of symmetry and vertex from the general form of its equation
 - Identify the axis of symmetry and vertex of a parabola from its equation in standard form
 - Graphs of Quadratic Functions
 - Write the equation of a quadratic function given vertex and a point on a graph
 - Write the equation of a quadratic function given intercepts on a graph
 - Write the equation of a quadratic function in standard form given the equation in general form
 - Applications of Quadratic Functions
 - Find the domain and range of a quadratic function
 - Determine the maximum and minimum values of quadratic functions
-

- Find the x- and y-intercepts of a quadratic function
- Use a quadratic function to model projectile motion
- Quadratic Regressions
 - Find the parabola of best fit using a graphing utility
 - Use a parabola of best fit to make predictions

5.2 Graphs of Polynomial and Power Functions

- End Behavior of Polynomial Functions
 - Identify power functions and polynomial functions
 - Identify if a graph is a polynomial function
 - Determine end behavior
- Local Behavior of Polynomial Functions
 - Identify intercepts of polynomial functions in factored form
 - Understand the relationship between degree, turning points, and x-intercepts
 - Understand the intermediate value theorem
 - Use factoring to find zeros of polynomial functions
 - Identify zeros and their multiplicities from an equation or a graph
- Write and Graph Polynomial Functions
 - Draw conclusions about a polynomial function from a graph
 - Graph polynomial functions
 - Write a formula for a polynomial function from a graph
 - Determine equation of a polynomial given key information

5.3 Dividing Polynomials

- Long Division of Polynomials
 - Use long division to divide polynomials
 - Use polynomial division to solve application problems
- Synthetic Division and Remainder Theorem
 - Use synthetic division to divide polynomials
 - Evaluate a polynomial using the remainder theorem

5.4 Zeros of Polynomial Functions

- Rational Zeros of Polynomial Functions
 - Use the factor theorem to solve a polynomial equation
 - Use the rational zero theorem to find rational zeros
 - Solve real-world applications of polynomial equations
- Complex Zeros of Polynomial Functions
 - Find zeros of polynomial functions with complex zeros
 - Use the linear factorization theorem to find polynomials with given zeros
 - Use Descartes' rule of signs

5.5 Rational Functions

- Asymptotic Behavior of Rational Functions
 - Identify removable discontinuities of rational functions
 - Identify slant asymptotes of rational functions
 - Use arrow notation to describe local behavior and end behavior of rational functions
 - Identify vertical asymptotes of rational functions
 - Identify horizontal asymptotes of rational functions
 - Graphs and Applications of Rational Functions
-

- Solve applied problems involving rational functions
- Find the intercepts of a rational function
- Graph rational functions
- Find the equation of a rational function from a graph

5.6 Inverses and Radical Functions

- Inverses of Polynomial Functions
 - Find the inverse of an invertible polynomial function
 - Restrict the domain to find the inverse of a polynomial function
 - Solve an application with the inverse of a function
- Inverses of Radical and Rational Functions
 - Find the inverse of a square root function
 - Find the inverse of a cube root function
 - Find the value of an inverse radical function
 - Find the domain of a radical function composed with a rational function
 - Find the inverse of a rational function

5.7 Modeling Using Variation

- Direct and Inverse Variation
 - Solve direct variation problems
 - Solve inverse variation problems
 - Solve problems involving joint variation

Chapter 6: Exponential and Logarithmic Functions

6.1 Exponential Functions

- Evaluate and Write Exponential Functions
 - Identify exponential functions
 - Evaluate exponential functions
 - Find the equation of an exponential function given the initial value and a point
 - Find the equation of an exponential function when the initial value is not known
- Applications of Exponential Functions and Base e
 - Find the equation of an exponential function in a word problem context
 - Calculate compound interest
 - Evaluate exponential functions with base e
 - Calculate continuous growth and decay

6.2 Graphs of Exponential Functions

- Exponential Function Graphs
 - Graph exponential functions
 - Graph exponential functions using transformations
 - Find the equation of an exponential function given a graph
 - Write an exponential function from a description

6.3 Logarithmic Functions

- Relate Logarithms and Exponents
 - Convert from logarithmic to exponential form
 - Convert from exponential to logarithmic form
 - Evaluate Logarithmic Expressions
 - Evaluate logarithms with positive integer solutions
-

- Evaluate logarithms with negative integer solutions
- Use common logarithms
- Use natural logarithms

6.4 Graphs of Logarithmic Functions

- Logarithmic Function Graphs
 - Identify the domain of a logarithmic function
 - Graph logarithmic functions
 - Graph transformations of logarithmic functions
 - Write a logarithmic function from a description

6.5 Logarithmic Properties

- Basic Properties of Logarithms
 - Understand the basic properties of logarithms
 - Use the product rule for logarithms
 - Use the quotient rule for logarithms
 - Use the power rule for logarithms
- Rewrite Logarithmic Expressions Using Properties
 - Expand logarithmic expressions
 - Condense logarithmic expressions
 - Use the change-of-base formula for logarithms

6.6 Exponential and Logarithmic Equations

- Solve Exponential Equations
 - Use like bases to solve exponential equations
 - Rewrite equations so all powers have the same base and solve exponential equations
 - Use logarithms to solve exponential equations
 - Solve an equation with a base e using natural logarithms
- Solve Logarithmic Equations
 - Use the definition of a logarithm to solve logarithmic equations
 - Use logarithm properties and the definition of the logarithm to solve logarithmic equations
 - Use the one-to-one property of logarithms to solve logarithmic equations

6.7 Exponential and Logarithmic Models

- Applications of Exponential and Logarithmic Functions
 - Model exponential growth
 - Model exponential decay
 - Applied logarithmic models
 - Choose an appropriate model for data
 - Express an exponential model in base e

6.8 Fitting Exponential Models to Data

- Exponential and Logarithmic Regressions
 - Build an exponential model from data using a graphing utility
 - Build a logarithmic model from data using a graphing utility
 - Logistic Growth Models
 - Use logistic growth models
 - Build a logistic model from data using a graphing utility
-

Chapter 7: Right Triangle Trigonometry and the Unit Circle

7.1 Vocabulary of Angles and Triangles

- Types of Angles
 - Identify right, acute, obtuse, and straight angles
 - Understand supplementary and complementary angles
- Angles, Triangles, and the Pythagorean Theorem
 - Find the measures of angles of a triangle using properties
 - Use properties of similar triangles to solve for a missing side

7.2 Angles as Rotations and Arc Length

- Angles as Rotations and Radian Measures
 - Identify the measure of positive and negative angles in standard position and the quadrant of the terminal side
 - Convert between degree and radian measure of an angle
 - Understand when two angles are coterminal
- Arc Length and Area of a Sector
 - Find the length of an arc
 - Find the area of a sector of a circle
 - Understand the relationship between linear and angular speed

7.3 The Unit Circle

- Sine and Cosine Values in the First Quadrant
 - Understand sine and cosine values on the unit circle
 - Find exact sine and cosine values for angles in the first quadrant of the unit circle
- Sine and Cosine Values with Reference Angles
 - Find the reference angle for a given angle
 - Use reference angles to evaluate sine and cosine functions
 - Use reference angles to find coordinates on the unit circle
 - Evaluate sine and cosine functions with a calculator

7.4 The Other Trigonometric Functions

- The Other Trigonometric Ratios on the Unit Circle
 - Find the secant, cosecant, tangent, and cotangent values for angles in the first quadrant of the unit circle
 - Use reference angles to evaluate secant, cosecant, tangent, and cotangent functions
 - Evaluate trigonometric functions with a calculator
- Use Given Trigonometric Ratios to Find Other Ratios
 - Understand the relationship between the quadrant in which an angle falls and the signs of the trig functions of that angle
 - Use the Pythagorean identity
 - Find the values of all trigonometric functions given coordinates on a unit circle
 - Find the values of all trigonometric functions given the value of one trigonometric function

Chapter 8: Periodic Functions

8.1 Sine and Cosine Graphs

- Characteristics of Sine and Cosine Graphs
 - Graph the sine function and understand its properties
-

- Graph the cosine function and understand its properties
- Transformations of Sine and Cosine Graphs
 - Determine the period and amplitude of a sinusoidal function
 - Determine the phase shift and vertical shift of a sinusoidal function
- Graph Sine and Cosine Functions
 - Find the equation of a sinusoidal function given a graph
 - Find the graph of a sinusoidal function given equation
 - Use sinusoidal functions to solve real-world applications

8.2 Graphs of Other Trigonometric Functions

- Characteristics of Tangent and Cotangent Graphs
 - Graph tangent functions
 - Graph cotangent functions
- Transformations of Tangent and Cotangent Functions
 - Graph tangent or cotangent functions over different periods
 - Graph transformations of tangent and cotangent functions
- Characteristics of Secant and Cosecant Graphs
 - Graph cosecant functions
 - Graph secant functions
- Graph Secant and Cosecant Functions
 - Graph transformations of cosecant functions
 - Graph transformations of secant functions
 - Find the equation of a cosecant function from a graph
 - Find the equation of a secant function from a graph

8.3 Inverse Trigonometric Functions

- Introduction to Inverse Trigonometric Functions
 - Understand inverse sine, cosine, and tangent functions
 - Understand inverse secant, cosecant, and cotangent functions
 - Use a calculator to evaluate inverse trigonometric functions
- Solve Triangles with Inverse Trigonometric Functions
 - Find an angle given two sides of a right triangle
 - Solve right triangle problems
- Compose Functions with Inverse Trigonometric Functions
 - Evaluate composite functions with inverse trigonometric functions in the form $f(f^{-1}(x))$ or $f(g^{-1}(x))$
 - Evaluate composite functions with inverse trigonometric functions in the form $f^{-1}(f(x))$ or $f^{-1}(g(x))$

Chapter 9: Trigonometric Identities and Equations

9.1 Fundamental Trigonometric Identities

- Simplify Expressions with Basic Trigonometric Identities
 - Understand quotient and reciprocal identities
 - Use even and odd identities in simplifying trigonometric expressions
 - Use Pythagorean and Cofunction Identities
 - Understand all forms of the Pythagorean identity
 - Use the cofunction identities
-

- Verify Trigonometric Identities
 - Use all identities to simplify trigonometric expressions
 - Use algebraic techniques to simplify trigonometric expressions
- 9.2 Double-Angle, Half-Angle, and Reduction Formulas
 - Double-Angle Formulas
 - Use double-angle formulas to find values of trigonometric functions
 - Use double-angle formulas to simplify trigonometric expressions
 - Half-Angle and Power-Reduction Formulas
 - Use reduction formulas to simplify an expression
 - Use half-angle formulas to find values of trigonometric functions
- 9.3 Sum-to-Product and Product-to-Sum Formulas
 - Sum-to-Product and Product-to-Sum Formulas
 - Express the product of trigonometric functions as a sum
 - Express sums of trigonometric functions as a product
- 9.4 Trigonometric Equations
 - Trigonometric Equations in Sine and Cosine
 - Solve linear trigonometric equations in sine
 - Solve linear trigonometric equations in cosine
 - Trigonometric Equations Involving a Single Trigonometric Function
 - Solve linear trigonometric equations in secant and cosecant
 - Solve linear trigonometric equations in tangent and cotangent
 - Solve trigonometric equations using a calculator
 - Trigonometric Equations in Quadratic Form or Requiring Factoring
 - Solve trigonometric equations in quadratic form requiring the quadratic formula
 - Solve factorable trigonometric equations in quadratic form
 - Solve equations with a single trigonometric function
 - Trigonometric Equations Requiring Identities or Multiple Angles
 - Solve trigonometric equations using cofunction identities
 - Solve trigonometric equations with multiple angles
 - Solve trigonometric equations using fundamental identities

Chapter 10: Further Applications of Trigonometry

10.1 Non-right Triangles - Law of Sines

- Law of Sines
 - Solve applied problems with the law of sines
 - Use the law of sines to solve SSA triangles
 - Use the law of sines to solve ASA or AAS triangles

10.2 Non-right Triangles - Law of Cosines and Area of Oblique Triangles

- Law of Cosines
 - Solve applied problems with the law of cosines
 - Use the law of cosines to solve SSS triangles
 - Use the law of cosines to solve SAS triangles
 - Area of Oblique Triangles
 - Use Heron's formula to find the area of a triangle
 - Find the area of an oblique triangle using the sine function
-

10.3 Polar Form of Complex Numbers

- Write Complex Numbers in Polar Form
 - Convert a complex number from polar to rectangular form
 - Write complex numbers in polar form
 - Find the absolute value of a complex number
 - Plot complex numbers
- Product and Quotient of Complex Numbers in Polar Form
 - Find the quotient of complex numbers in polar form
 - Find the product of complex numbers in polar form
- Powers and Roots of Complex Numbers in Polar Form
 - Find roots of complex numbers in polar form
 - Find powers of complex numbers in polar form

10.4 Parametric Equations

- Write Parametric Equations
 - Find the parametric equations for a line segment given an orientation
 - Parameterize a curve
- Eliminate the Parameter
 - Eliminate the parameter in trigonometric parametric equations
 - Eliminate the parameter in exponential and logarithmic equations
 - Eliminate the parameter in polynomial and radical equations
 - Eliminate the parameter in linear equations

10.5 Graphs with Parametric Equations

- Graph Parametric Equations
 - Use parametric equations in applications
 - Graph trigonometric parametric equations by plotting points
 - Graph parametric equations by plotting points

Chapter 11: Systems of Equations and Inequalities

11.1 Systems of Linear Equations in Two Variables

- Graphing Systems of Linear Equations
 - Solve systems of equations in two variables by graphing
 - Determine whether an ordered pair is a solution to a system of equations
- Solving Systems of Linear Equations
 - Identify inconsistent and dependent systems of equations containing two variables, and express the solution of dependent equations
 - Solve systems of equations in two variables by addition
 - Solve systems of equations in two variables by substitution
- Applications of Systems of Linear Equations
 - Write and solve a system of equations in two variables from a word problem
 - Use systems of equations to investigate profits
- Linear Inequalities in Two Variables
 - Solve a linear system of inequalities by graphing
 - Solve a linear inequality in two variables by graphing

11.2 Systems of Nonlinear Equations in Two Variables

- Systems of Two Nonlinear Equations
-

- Solve a system of nonlinear equations in two variables using elimination
- Solve a system of nonlinear equations representing a circle and a line
- Solve a system of nonlinear equations representing a parabola and a line
- Graphing Nonlinear Inequalities and Systems of Inequalities
 - Graph a system of nonlinear inequalities
 - Graph a nonlinear inequality

11.3 Partial Fractions

- Partial Fraction Decomposition with Linear Factors
 - Decompose a rational expression where the denominator has repeated linear factors
 - Decompose a rational expression where the denominator has only non-repeated linear factors
- Partial Fraction Decomposition with Quadratic Factors
 - Decompose a rational expression where the denominator has a repeated irreducible quadratic factor
 - Decompose a rational expression where the denominator has a non-repeated irreducible quadratic factor

Chapter 12: Conic Sections

12.1 Ellipses

- Ellipses Centered at the Origin
 - Graph an ellipse centered at the origin from an equation in standard form
 - Write the equation in standard form of an ellipse centered at the origin
 - Identify key points and axes of ellipses from an equation
 - Identify key points and axes of ellipses from a graph
- Ellipses Not Centered at the Origin
 - Graph an ellipse not centered at the origin
 - Write the equation in standard form of an ellipse not centered at the origin
 - Identify key points and axes of ellipses not centered at the origin
- Ellipses Not in Standard Form and Applications of Ellipses
 - Use ellipses in applications
 - Graph an ellipse where the equation is not given in standard form
 - Convert an equation of an ellipse into standard form

12.2 Hyperbolas

- Hyperbolas Centered at the Origin
 - Graph a hyperbola centered at the origin from an equation in standard form
 - Write the equation of a hyperbola centered at the origin in standard form
 - Identify vertices, foci, and asymptotes of a hyperbola from an equation
 - Locate the vertices and foci of a hyperbola from a graph
 - Hyperbolas Not Centered at the Origin
 - Graph a hyperbola not centered at the origin from an equation in standard form
 - Write the equation of a hyperbola not centered at the origin
 - Identify vertices, foci, and asymptotes of a hyperbola not centered at the origin
 - Hyperbolas Not in Standard Form and Applications of Hyperbolas
 - Use hyperbolas in applications
 - Graph a hyperbola from an equation given in general form
-

- Convert an equation of a hyperbola into standard form

12.3 Parabolas

- Parabolas with Vertices at the Origin
 - Write the equation of a parabola with its vertex at the origin in standard form
 - Graph a parabola with its vertex at the origin
 - Identify key components of a parabola from an equation
 - Identify key components of a parabola from a graph
- Parabolas with Vertices Not at the Origin
 - Write the equation of a parabola with its vertex not at the origin in standard form
 - Graph a parabola with its vertex not at the origin
 - Identify key components of a parabola with its vertex not at the origin
- Parabolas Not in Standard Form and Applications of Parabolas
 - Use parabolas in applications
 - Graph a parabola from an equation given in general form
 - Convert an equation of a parabola into standard form

12.4 Rotation of Axes

- Conics in General Form and Rotation of Conics
 - Find the location of a point after a rotation of axes
 - Identify a conic from its general form

12.5 Conic Sections in Polar Coordinates

- Conic Sections in Polar Coordinates
 - Find the polar equation of a conic given its focus, eccentricity, and directrix
 - Identify the type of conic from its polar equation

Chapter 13: Limits and Continuity

13.1 Finding Limits Using Numerical and Graphical Approaches

- Limits From a Graph or Table
 - Understand the properties of limits
 - Evaluate limits graphically
 - Understand the limit of a function and evaluate a limit from a table

13.2 Finding Limits Analytically

- Limits Analytically for Continuous and Piecewise Functions
 - Evaluate limits analytically for absolute value functions
 - Evaluate limits analytically for piecewise functions
 - Evaluate two-sided limits analytically for continuous functions
 - Limits Analytically for Functions with Removable Discontinuities
 - Evaluate two-sided limits analytically for rational functions that contain radicals with removable discontinuities
 - Evaluate two-sided limits analytically for complex fractions with removable discontinuities
 - Evaluate two-sided limits analytically for rational functions with removable discontinuities through expansion
 - Evaluate two-sided limits analytically for rational functions with removable discontinuities by factoring
 - Limits Analytically for Trigonometric Functions
 - Evaluate trigonometric limits using identities
-

- Evaluate trigonometric limits using special limits
- Evaluate trigonometric limits using direct substitution
- Infinite Limits
 - Evaluate limits analytically for trigonometric functions with essential discontinuities
 - Evaluate limits analytically for functions with essential discontinuities
- Limits at Infinity
 - Evaluate limits of radical and exponential functions at infinity
 - Evaluate limits of trigonometric functions at infinity
 - Evaluate limits of rational functions at infinity
 - Evaluate limits of polynomial functions at infinity

13.3 Derivatives

- The Average Rate of Change
 - Find the average rate of change given a function and variable intervals
 - Find the average rate of change given a function
 - The Derivative of Function
 - Use the limit definition to find the derivative of a function with a radical
 - Use the limit definition to find the derivative of a rational function
 - Use the limit definition to find the derivative of a polynomial function
 - Find the derivative of a function at a point using limits
 - Instantaneous Rates of Change
 - Use instantaneous rates of change in applications
 - Estimate the derivative at a point on the graph of a function
 - Determine the sign of the slope of a line tangent to a function at a given point
 - Differentiability
 - Determine where a function is differentiable from a graph
-