



Finite Mathematics and Calculus

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Knewton Finite Math with Business Calculus was developed to meet the scope and sequence of a typical one semester Finite Math with Calculus course. To develop the course, Knewton used a variety of different source content, including OpenStax Calculus, a Math in Society textbook developed by a professor at Pierce College and the Open Course Library project, videos created by a Math Professor we have partnered with, and a team of Subject Matter Experts (SMEs). The SMEs come from diverse backgrounds and are all accomplished academics in the field of Mathematics.

Knewton Finite Math with Business Calculus has two instructional sequences for every learning objective, giving students multiple opportunities to learn new concepts. Between our text, video, and Knewton SMEs, we were able to solicit ideas from math instructors and students at all levels of higher education. Knewton Finite Math with Business Calculus covers the typical breadth of topics and also provides the necessary depth to ensure the course is manageable and engaging for instructors and students alike.

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- The Fundamental Counting Principle
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- Volume Using the Disk Method
 - Use the disk method to find the volume of a solid of revolution around the x-axis with polynomials or roots
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Applications in Business and Economics

- Income Streams
 - Find the total income for a continuous stream
 - Find the present value of a continuous stream
 - Find the future value of a continuous stream
- Consumer and Producer Surplus
 - Find the consumers' surplus at a certain price level
 - Find the producers' surplus at a certain price level
 - Find the equilibrium price and the consumers' and producers' surplus at that price

Integration by Parts

- Basic Integration by Parts with Indefinite Integrals
 - Use integration by parts when u and v are given
 - Use integration by parts for indefinite integrals

Integration Using Tables

- Integration Tables
 - Use a formula from an integration table to evaluate an integral
- Integration with Computer Algebra Systems
 - Use a computer algebra system to evaluate an integral
- Reduction Formulas
 - Use a table with reduction formulas to evaluate an integral

Chapter 18: Multivariable Calculus

Functions of Several Variables

- Introduction to Multivariable Functions
 - Evaluate a multivariable function
 - Sketch a point in three-dimensional space
 - Graph cross sections of a multivariable function

Partial Derivatives

- Partial Derivatives of a Function of Two Variables
 - Find the partial derivative of a function of two variables
 - Estimate the partial derivative of a function at a point from a graph or contour map
 - Total Differential
 - Use the differential to approximate the change in a function given the change in the inputs or to calculate maximum error
 - Partial Derivatives of a Function of Three or More Variables
 - Find the partial derivative of a function of three variables
 - Higher Order Partial Derivatives
 - Find the higher order partial derivatives of a function of two variables
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- The Chain Rule for Functions of Several Variables
 - Use the chain rule for one independent variable
 - Use the chain rule for two independent variables
 - Use the generalized chain rule

Maxima and Minima

- Critical Points and the Second Derivative Test for Functions of Two Variables
 - Find critical points of a function of two variables
 - Use the second derivative test to classify critical points of a function of two variables
- Absolute Extrema and Applications for Functions of Two Variables
 - Find the absolute extrema of a function of two variables on a closed region
 - Solve maximization and minimization word problems with multiple variables

Maxima and Minima Using Lagrange Multipliers

- Lagrange Multipliers with One Constraint
 - Use Lagrange multipliers to find maximum and minimum values of a function of two variables with a single constraint
 - Use Lagrange multipliers to find maximum and minimum values of a function of three variables with a single constraint
- Lagrange Multipliers with Two Constraints
 - Use Lagrange multipliers to find maximum and minimum values of a function with two constraints

Double Integrals over Rectangular Regions

- Iterated Integrals and Properties of Double Integrals
 - Recognize and use some of the properties of double integrals
 - Evaluate a double integral over a rectangular region by writing it as an iterated integral
 - Evaluate a double integral over a rectangular region by reversing the order of integration
- Applications of Double Integrals Over Rectangular Regions
 - Find the volume under a surface
 - Find the average value of a function over a rectangular region

Double Integrals over More General Regions

- Double Integrals Over Nonrectangular Regions
 - Recognize when a function of two variables is integrable over a general region
 - Evaluate a double integral by computing an iterated integral over a region bounded by two lines and two functions
 - Double Integrals by Decomposing Regions or Changing the Order of Integration
 - Evaluate a double integral over a more complex region by decomposing the region
 - Simplify the calculation of an iterated integral by changing the order of integration
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- Applications of Double Integrals Over General Regions
 - Use double integrals to calculate the area of a general plane region
 - Use double integrals to calculate the volume of a region between two surfaces over a general plane region
 - Find the average value of a function over a general region

Chapter 19: Trigonometric Functions

Trigonometric Functions Review

- Angles as Rotations and Radian Measures
 - Identify the measure of positive and negative angles in standard position and the quadrant of the terminal side
 - Convert between degree and radian measure of an angle
 - Understand when two angles are coterminal
 - The Six Trigonometric Ratios
 - Use right triangles to evaluate sin, cos, and tan functions
 - Evaluate reciprocal trig functions using right triangles or a sin, cos, or tan function
 - Sine and Cosine Values in the First Quadrant
 - Understand sin and cos values on the unit circle
 - Find exact sin and cos values for angles in the first quadrant of the unit circle
 - Sine and Cosine Values with Reference Angles and a Calculator
 - Find the reference angle for a given angle
 - Use reference angles to evaluate sin and cos functions
 - Use reference angles to find coordinates on the unit circle
 - Evaluate sin and cos functions with a calculator
 - The Other Trigonometric Ratios on the Unit Circle
 - Find the sec, csc, tan, and cot values for angles in the first quadrant of the unit circle
 - Use reference angles to evaluate sec, csc, tan, and cot functions
 - Evaluate trigonometric functions with a calculator
 - Use Given Trigonometric Ratios to Find Other Ratios
 - Understand the relationship between the quadrant in which an angle falls and the signs of the trig functions of that angle
 - Use the pythagorean identity
 - Find the values of all trigonometric functions given coordinates on a unit circle
 - Find the values of all trigonometric functions given the value of one trigonometric function
 - Characteristics of Sin and Cos Graphs
 - Graph the sin function and understand its properties
 - Graph the cos function and understand its properties
 - Applications of Trigonometric Functions
 - Use sinusoidal functions to solve real-world applications
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Derivatives of Trigonometric Functions

- Derivatives with Trigonometric Functions
 - Find the derivative of a function with sine or cosine
 - Use the product or quotient rule to find a derivative with sine or cosine
 - Use the chain rule with trigonometric functions
- Applications of Trigonometric Derivatives
 - Compute derivatives of trigonometric functions in application problems

Integration of Trigonometric Functions

- Integration with Trigonometric Functions
 - Evaluate indefinite integrals involving trigonometric functions
 - Evaluate definite integrals involving trigonometric functions
 - Compute integrals of trigonometric functions in application problems

Chapter 20: Differential Equations

Separation of Variables

- Finding Differential Equation Solutions using Separation of Variables
 - Find a general solution to a differential equation composed of two polynomials using separation of variables
 - Find a specific solution to a differential equation composed of two polynomials using separation of variables
- Application Problems using Separation of Variables
 - Solve solution concentration problems using separation of variables
 - Solve Newton's law of cooling problems using separation of variables

First-Order Linear Differential Equations

- Recognizing and Solving First-order Linear Differential Equations
 - Identify first-order linear differential equations
 - Write first-order linear differential equations in standard form
 - Solve a first-order linear differential equation using an integrating factor
- Applications of First-order Linear Differential Equations
 - Solve first-order differential equation problems about compound interest
 - Solve first-order differential equation problems involving equilibrium price

Chapter 21: Taylor Polynomials and Sequences and Series

Taylor Polynomials

- Taylor and Maclaurin Polynomials
 - Recognize a Taylor series
 - Find the Taylor polynomials for a function at a value
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Taylor Series

- Representing Functions with Taylor and Maclaurin Series
 - Represent a function at a value with a Taylor series and determine the interval of convergence
 - Find the Maclaurin series for a function and show that the series converges

Operations on Taylor Series

- Finding the Maclaurin Series that Represents a Function
 - Find the Maclaurin series for a trigonometric function
 - Find the Maclaurin series for a logarithmic or exponential function
 - Find a Maclaurin series by differentiating another series

Approximations Using Taylor Series

- Estimating Function Values with Taylor and Maclaurin Series
 - Find the Maclaurin polynomials for a function
 - Estimate a function value using Taylor polynomials
 - Determine the error of an estimated function value using Taylor's theorem
 - Estimate a trigonometric function value using Maclaurin polynomials
- Alternating Series and the Alternating Series Test
 - Determine if an alternating series converges or diverges using the alternating series test
 - Estimate the remainder of an alternating series
 - Determine whether a series converges absolutely or conditionally

Sequences and Series

- Geometric Sequences
 - Find the common ratio of a geometric sequence
 - Write terms of a geometric sequence
 - Write a recursive formula for a geometric sequence
 - Write an explicit formula for a geometric sequence
 - Applications of Series
 - Solve application problems with arithmetic series
 - Solve application problems with geometric series
 - Find the equivalent fraction for a repeating decimal
 - Solve an annuity problem
 - Finite and Infinite Geometric Series
 - Find the sum of a finite geometric series
 - Determine if the sum of an infinite series is defined
 - Find the sum of an infinite geometric series
 - Newton's Method
 - Use Newton's method to approximate the root of a polynomial
 - Use Newton's method to approximate a square root
 - Determine when Newton's method does not work
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Chapter 22: Probability and Calculus

Improper Integrals

- Improper Integrals over Infinite Intervals
 - Evaluate an improper integral over an infinite interval
 - Evaluate an improper integral from negative infinity to positive infinity

Continuous Random Variables

- Probability Density Functions
 - Understand the properties of probability density functions
 - Compute probability using a continuous probability density function
- Cumulative Distribution Functions
 - Understand the properties of a cumulative distribution function
 - Find the cumulative distribution function given a probability density function

Parameters of Continuous Random Variables

- Mean and Median of a Probability Density Function
 - Find the mean of a continuous probability density function
 - Find the median of a continuous probability density function
- Other Parameters of Probability Density Functions
 - Find the variance and standard deviation of a continuous probability density function

Special Probability Distributions

- Uniform Distribution
 - Compute probability using the uniform distribution
 - Compute the mean, median, and standard deviation of the uniform distribution
 - Exponential Distribution
 - Compute probability using the exponential distribution
 - Compute the mean, median, and standard deviation of the exponential distribution
 - Normal Distribution
 - Understand the notation and interpret the parameters of a normal distribution
 - Compute z-scores and use them to compare values from different data sets
 - Use a table to find probabilities in a normal distribution
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