



# Calculus I

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Knewton Single Variable Calculus was developed to meet the scope and sequence of a typical two semester calculus course . To develop the course, Knewton used three main sources of content: Openstax, videos created by a Math Professor we have partnered with, and a team of Subject Matter Experts. The SMEs come from diverse backgrounds and are all academics in the field of Mathematics.

Knewton Single Variable Calculus has two instructional sequences for every learning objective, giving students multiple opportunities to learn new concepts. Between our OpenStax text content, instructional videos, and Knewton SMEs, we were able to solicit ideas from math instructors and students. Knewton Single Variable Calculus covers the typical breadth of calculus topics and also provides the necessary depth to ensure the course is manageable and engaging for instructors and students alike.

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  - Eliminate the parameter in exponential and logarithmic equations
  - Eliminate the parameter in trigonometric parametric equations
- Graph Parametric Equations
  - Graph parametric equations by plotting points
  - Graph trigonometric parametric equations by plotting points
  - Use parametric equations in applications
- Cycloids
  - Graph a cycloid defined by parametric equations
  - Determine the number of cusps on a hypocycloid

#### Calculus of Parametric Curves

- Derivatives of Parametric Equations
  - Find the derivative of a curve defined by polynomial parametric equations
  - Find the derivative of a curve defined by trigonometric parametric equations
  - Find the equation of a line tangent to a parametrically defined curve
  - Find the second derivative of curve defined by parametric equations
- Integrating Parametric Curves
  - Find the area under a curve defined by parametric equations
  - Find the arc length of a curve defined by trigonometric parametric equations
  - Find the surface area of a volume of revolution generated by revolving a parametrically defined curve

#### Calculus in Polar Coordinates

- Area and Arc Length in Polar Coordinates
  - Find the area of a region between two polar curves
  - Find the arc length of a polar curve
  - Find the area of a region bounded by a polar curve

#### Conic Sections

- Ellipses Centered at the Origin
    - Identify key points and axes of ellipses from a graph
    - Identify key points and axes of ellipses from an equation
    - Write the equation in standard form of an ellipse centered at the origin
    - Graph an ellipse centered at the origin from an equation in standard form
  - Ellipses Not Centered at the Origin
    - Identify key points and axes of ellipses not centered at the origin
    - Write the equation in standard form of an ellipse not centered at the origin
    - Graph an ellipse not centered at the origin
  - Ellipses Not in Standard Form and Applications of Ellipses
    - Convert an equation of an ellipse into standard form
    - Graph an ellipse where the equation is not given in standard form
    - Use ellipses in applications
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- Hyperbolas Centered at the Origin
  - Locate the vertices and foci of a hyperbola from a graph
  - Identify vertices, foci, and asymptotes of a hyperbola from an equation
  - Write the equation of a hyperbola centered at the origin in standard form
  - Graph a hyperbola centered at the origin from an equation in standard form
- Hyperbolas Not Centered at the Origin
  - Identify vertices, foci, and asymptotes of a hyperbola not centered at the origin
  - Write the equation of a hyperbola not centered at the origin
  - Graph a hyperbola not centered at the origin from an equation in standard form
- Hyperbolas Not in Standard Form and Applications of Hyperbolas
  - Convert an equation of a hyperbola into standard form
  - Graph a hyperbola from an equation given in general form
  - Use hyperbolas in applications
- Parabolas Centered at the Origin
  - Identify key components of a parabola from a graph
  - Identify key components of a parabola from an equation
  - Graph a parabola centered at the origin
  - Write the equation of a parabola centered at the origin in standard form
- Parabolas Not Centered at the Origin
  - Identify key components of a parabola not centered at the origin
  - Graph a parabola not centered at the origin
  - Write the equation of a parabola not centered at the origin in standard form
- Parabolas Not in Standard Form and Applications of Parabolas
  - Convert an equation of a parabola into standard form
  - Graph a parabola from an equation given in general form
  - Use parabolas in applications

## **Algebra Review: Functions and Graphs**

### Review of Functions

- Functions, Function Notation, and Domain and Range
  - Evaluate a function using function notation and determine the domain and range
  - Represent functions using tables, graphs, or formulas
- Symmetry of Functions, Absolute Value, and Function Composition
  - Combine functions using mathematical operators or function composition
  - Understand the symmetry of functions and the absolute value function

### Basic Classes of Functions

- Graphing Lines, Parabolas, and Polynomials
    - Find the slope and equation of lines
    - Use tools such as the quadratic formula and end behavior to graph polynomial functions
    - Understand the difference between algebraic and transcendental functions and find the domain of algebraic functions
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- Piecewise Functions and Transformations of Functions
  - Understand and graph piecewise-defined functions
  - Perform transformations of functions

#### Trigonometric Functions

- Radian Measure and Evaluating Trigonometric Functions
  - Understand radian measure and convert between radians and degrees
  - Evaluate trigonometric functions
- Trigonometric Identities, Equations, and Graphs
  - Understand trigonometric identities and use them to solve trigonometric equations
  - Graph periodic functions

#### Inverse Functions

- Finding an Inverse Graphically and Algebraically
  - Determine when a function is one-to-one
  - Find the inverse of a function
  - Graph an inverse function
  - Restrict the domain of a function to find an inverse function
- Inverse Trigonometric Functions
  - Evaluate expressions involving inverse trigonometric functions

#### Exponential and Logarithmic Functions

- Evaluating and Graphing Exponential Functions
    - Identify and evaluate exponential functions
    - Graph an exponential function
    - Simplify expressions using the law of exponents
    - Understand the base  $e$  and use it for application problems
  - Logarithmic Graphs and Properties of Logarithms
    - Convert between logarithmic and exponential form and evaluate logarithmic expressions
    - Graph a logarithmic function
    - Understand the properties of logarithms
  - Exponential and Logarithmic Equations
    - Solve equations involving exponential functions
    - Solve equations involving logarithmic functions
    - Use the change-of-base formula to evaluate logarithms
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